Answer on Question #46193, Math, Calculus

The limit $\lim_{x\to 0^+} \frac{\ln \tan 2x}{\ln \tan x}$ has uncertainty $\frac{\infty}{\infty}$.

Let us use L'Hopitals rule in order to evaluate this limit:

$$\lim_{x\to 0^+} \frac{\ln \tan 2x}{\ln \tan x} = \lim_{x\to 0^+} \frac{\frac{1}{\tan 2x} \cdot \frac{1}{\cos^2 2x} \cdot 2}{\frac{1}{\tan x} \cdot \frac{1}{\cos^2 x}} = \lim_{x\to 0^+} \frac{2 \tan x \cos^2 x}{\tan 2x \cos^2 2x} =$$

$$= 2 \lim_{x\to 0^+} \frac{\sin x \cdot \cos x}{\sin 2x \cdot \cos 2x} = 2 \lim_{x\to 0^+} \frac{\frac{1}{2} \sin 2x}{\sin 2x \cos 2x} = \lim_{x\to 0^+} \frac{1}{\cos 2x} = 1 .$$

Therefore, $\lim_{x\to 0^+} \frac{\ln \tan 2x}{\ln \tan x} = 1$.