Answer on Question #46192 – Math – Calculus

Question. Evaluate the limit:

$$\lim_{x\to-\infty}\frac{\sin(1/x)}{e^{1/x}-1}$$

Solution. Notice that

$$\lim_{x\to-\infty}\sin(1/x)=\sin(0)=0,$$

and

$$\lim_{x\to-\infty}(e^{1/x}-1)=e^0-1=0$$

as well. Therefore we can use L'Hôpital's rule claiming, that if f and g are differentiable functinos at some point a, and

$$\lim_{x\to a}f(x)=\lim_{x\to a}g(x)=0,$$

then

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}.$$

Since

$$(\sin(1/x))'=\cos(1/x)\cdot(1/x)'=\cos(1/x)\cdot\frac{-1}{x^2}=-\frac{\cos(1/x)}{x^2},$$

and

$$\left(e^{1/x}-1\right)'=e^{1/x}\cdot(1/x)'=e^{1/x}\cdot\frac{-1}{x^2}=-\frac{e^{1/x}}{x^2},$$

we obtain that

$$\lim_{x\to-\infty}\frac{\sin(1/x)}{e^{1/x}-1}=\lim_{x\to-\infty}\frac{-\frac{\cos(1/x)}{x^2}}{-\frac{e^{1/x}}{x^2}}=\lim_{x\to-\infty}\frac{\cos(1/x)}{e^{1/x}}=\frac{\cos 0}{e^0}=\frac{1}{1}=1.$$

Answer. $\lim\limits_{x\to-\infty}\frac{\sin(1/x)}{e^{1/x}-1}=1.$