Answer on Question #46185 - Math - Vector Calculus

Find co-ordinates of the points on the line (x+2)/3= (y+1)/2= (z-3)/2 at a distance 3√2 from the point (1, 2, 3)

Solution.

$(x^{'}, y^{'}, z^{'})$ - point that we want to find.

Distance between points $(x^{'}, y^{'}, z^{'})$ and $(1,2,3)$:

$$\sqrt{(x^{'}-1)^2+(y^{'}-2)^2+(z^{'}-3)^2}=3\sqrt{2}$$

Then

$$(x^{'}-1)^2+(y^{'}-2)^2+(z^{'}-3)^2=18$$

Point $(x^{'}, y^{'}, z^{'})$ must satisfy the equation of the line, that why

$$\frac{x^{'}+2}{3}=\frac{y^{'}+1}{2}=\frac{z^{'}-3}{2}$$

So we have system of equations

$$\begin{cases}(x^{'}-1)^2+(y^{'}-2)^2+(z^{'}-3)^2=18\\ \frac{x^{'}+2}{3}=\frac{y^{'}+1}{2}=\frac{z^{'}-3}{2}\end{cases}$$

$$\frac{y^{'}+1}{2}=\frac{z^{'}-3}{2} \qquad z^{'}-3=y^{'}+1$$

$\frac{x^{'}+2}{3}=\frac{y^{'}+1}{2}$ $\qquad x^{'}+2=\frac{3(y^{'}+1)}{2}$ $\qquad x^{'}-1=\frac{3(y^{'}+1)}{2}-3=\frac{3}{2}(y^{'}-1)$

$$\begin{cases}(\frac{3}{2}(y^{'}-1))^2+(y^{'}-2)^2+(y^{'}+1)^2=18\\ \frac{x^{'}+2}{3}=\frac{y^{'}+1}{2}=\frac{z^{'}-3}{2}\end{cases}$$

$$\begin{cases}\frac{17}{4}y^{'2}-\frac{13}{2}y^{'}-\frac{43}{4}=0\\ \frac{x^{'}+2}{3}=\frac{y^{'}+1}{2}=\frac{z^{'}-3}{2}\end{cases}$$

$\frac{17}{4}y^{'2}-\frac{13}{2}y^{'}-\frac{43}{4}=0$

$17y^{'2}-26y^{'}-43=0$ $\quad D=26^2+4\cdot17\cdot43=3600$

$y_1^{'}=\frac{26+\sqrt{D}}{2\cdot17}=\frac{43}{17}$ $\qquad y_2^{'}=\frac{26-\sqrt{D}}{2\cdot17}=-1$

$x_1^{'} = \frac{3}{2}(y_1^{'} - 1) + 1 = \frac{56}{17}$ $\quad$ $x_2^{'} = \frac{3}{2}(y_2^{'} - 1) + 1 = -2$

$z_1^{'} = y_1^{'} + 4 = \frac{111}{17}$ $\quad$ $z_2^{'} = y_2^{'} + 4 = 3$

Our points : $\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$ and $(-2, -1, 3)$.

Answer: $\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$ and $(-2, -1, 3)$.