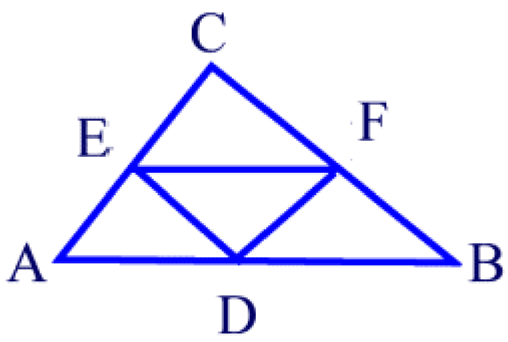
Answer on Question #46180 – Math – Geometry

Prove that in any triangle the line joining the mid-points of any two sides is parallel to the third side and half of its length.



Proof

Let AD = BD and AE = CE. Prove that DE||BC and DE = BC/2.

Extend DE beyond E to F such that DE = EF. Since AE = CE, triangles ADE and CEF are equal, making CF||AB (or CF||BD, which is the same) because, for the transversal AC, the alternating angles DAE and ECF are equal. Also, CF = AD = BD, such that BDFC is a parallelogram. It follows that BC = DF = 2·DE which is what we set out to prove.

Conversely, let D be on AB, E on AC, DE||BC and DE = BC/2. Prove that AD = DB andAE = CE.

This is so because the condition DE||BC makes triangles ADE and ABC similar, with implied proportion,

AB/AD = AC/AE = BC/DE = 2.

It thus follows that AB is twice as long as AD so that D is the midpoint of AB; similarly, E is the midpoint of AC.