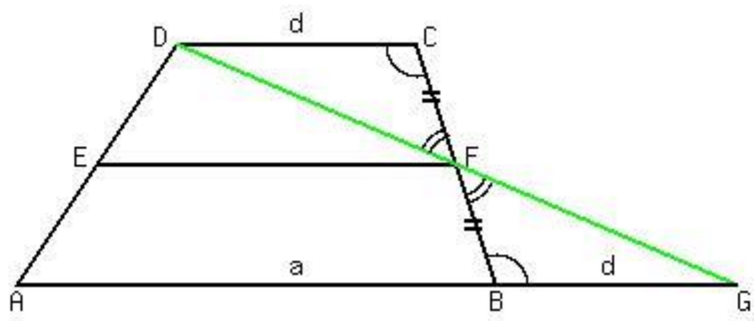
Answer on Question #46179 – Math – Geometry

Prove that the straight line joining the mid-points of two non-parallel sides of a trapezium is parallel to the parallel sides and half of their sum.

Solution



The segments FC and BF are congruent since the point F is the midpoint of the side BC. The angles DFC and BFG are congruent as the vertical angles. The angles DCF and FBG are congruent as the alternate exterior angles at the parallel lines AB and DC and the transverse . Hence, the triangles DFC and FBG are congruent in accordance with the ASA-test of congruency of triangles.

It implies that the segments DF and GF are congruent as the corresponding sides of the congruent triangles DFC and FBG.

Thus the mid-line EF of the trapezoid ABCD is the straight line segment connecting the midpoints of the triangle AGD.

It is well known fact that the the straight line segment connecting the midpoints of the triangle AGD is parallel to the triangle base AG and its length is half of the length of the triangle base. Therefore EF is parallel to AG, hence EF is parallel to CD, because it is given that AG and CD are parallel.

In our case, the length of the segment EF is half of the length AG :

$$|EF| = \frac{1}{2}|AG| = \frac{1}{2}(|AB| + |BG|).$$

Since $|BG| = |DC|$ **from the triangles congruency, we have** $|EF| = \frac{1}{2}|AG| = \frac{1}{2}(|AB| + |BG|)$**,**

or $|EF| = \frac{1}{2}(a + d)$, **where a and d are the lengths of the trapezoid bases.**