Answer on Question #46178 – Math - Vector Calculus

Problem.

The vectors □(→┬a) and □(→┬b) are non-collinear. Find for what value of x, the vectors □(→┬c)=(x-2) □(→┬a)+□(→┬b) and □(→┬d) =(2x+1) □(→┬a)- □(→┬b) are collinear.

Solution.

The vectors $\vec{c} = (x-2)\vec{a} + \vec{b}$ and $\vec{d} = (2x+1)\vec{a} - \vec{b}$ are colliner if there exists λ such that

$$\vec{c} = \lambda\vec{d}.$$

Then $(x-2)\vec{a} + \vec{b} = \lambda((2x+1)\vec{a} - \vec{b})$. Hence $\big((x-2) - \lambda(2x+1)\big)\vec{a} = (-1-\lambda)\vec{b}$. The vectors $\vec{a}$ and $\vec{b}$ are non-collinear, so $(x-2) - \lambda(2x+1) = 0$ and $-1-\lambda = 0$. Hence $\lambda = -1$ and $(x-2) + (2x+1) = 0$, $3x = 1$.

Therefore $x = \frac{1}{3}$.

Answer. $x = \frac{1}{3}$.