Answer on Question #46176 – Math – Vector Calculus

Problem.

Find the area of a parallelogram whose diagonals 2□(→┬m-) □(→┬n) and 4□(→┬m)- 5□(→┬n) , where □(→┬m) and □(→┬n) are unit vectors forming an angle of 45°.

Solution:

The magnitude of the cross product

$$(2\vec{m} - \vec{n}) \times (4\vec{m} - 5\vec{n})$$

equal to the area of the parallelogram that this vectors span.

$$(2\vec{m} - \vec{n}) \times (4\vec{m} - 5\vec{n}) = 2\vec{m} \times 4\vec{m} + 2\vec{m} \times (-5\vec{n}) + (-\vec{n}) \times 4\vec{m} + (-\vec{n}) \times (-5\vec{n})$$
$$= -10(\vec{m} \times \vec{n}) - 4(\vec{n} \times \vec{m}) = -10(\vec{m} \times \vec{n}) + 4(\vec{m} \times \vec{n}) = -6(\vec{m} \times \vec{n}),$$

as cross product of collinear vectors equals $\vec{0}$ and $\vec{m} \times \vec{n} = -\vec{n} \times \vec{m}$.

$$|-6(\vec{m} \times \vec{n})| = 6|\vec{m} \times \vec{n}| = 6|\vec{m}| \cdot |\vec{n}| \cdot \sin(\vec{m}, \vec{n}) = 6 \cdot 1 \cdot 1 \cdot \sin 45^\circ = \frac{6}{\sqrt{2}} = 3\sqrt{2}.$$

Answer: $3\sqrt{2}$.