

Answer on Question #46174 – Math – Complex Analysis

Find the square root of $4ab - 2i(a^2 - b^2)$.

$$\sqrt{4ab - 2i(a^2 - b^2)}$$

Solution

1) if $a > 0, b > 0$ or $a < 0, b < 0$

$$\sqrt{4ab - 2i(a^2 - b^2)} = \sqrt{2(a^2 + b^2)} \left(\cos\left(\frac{1}{2} \operatorname{arctg}\left(\frac{b^2 - a^2}{2ab}\right)\right) + i \sin\left(\frac{1}{2} \operatorname{arctg}\left(\frac{b^2 - a^2}{2ab}\right)\right) \right);$$

$$\begin{aligned} \sqrt{4ab - 2i(a^2 - b^2)} &= \sqrt{2(a^2 + b^2)} \left(\cos\left(\frac{1}{2} \operatorname{arctg}\left(\frac{b^2 - a^2}{2ab}\right) + \pi\right) \right. \\ &\quad \left. + i \sin\left(\frac{1}{2} \operatorname{arctg}\left(\frac{b^2 - a^2}{2ab}\right) + \pi\right) \right). \end{aligned}$$

2) if $(a < 0, b > 0$ or $a > 0, b < 0)$ with $b^2 > a^2$

$$\begin{aligned} \sqrt{4ab - 2i(a^2 - b^2)} &= \sqrt{2(a^2 + b^2)} \left(\cos\left(\frac{1}{2} \operatorname{arctg}\left(\frac{b^2 - a^2}{2ab}\right) + \frac{\pi}{2}\right) + \right. \\ &\quad \left. i \sin\left(\frac{1}{2} \operatorname{arctg}\left(\frac{b^2 - a^2}{2ab}\right) + \frac{\pi}{2}\right) \right); \end{aligned}$$

$$\begin{aligned} \sqrt{4ab - 2i(a^2 - b^2)} &= \sqrt{2(a^2 + b^2)} \left(\cos\left(\frac{1}{2} \operatorname{arctg}\left(\frac{b^2 - a^2}{2ab}\right) + \frac{3\pi}{2}\right) + \right. \\ &\quad \left. i \sin\left(\frac{1}{2} \operatorname{arctg}\left(\frac{b^2 - a^2}{2ab}\right) + \frac{3\pi}{2}\right) \right). \end{aligned}$$

3) if $(a < 0, b > 0$ or $a > 0, b < 0)$ with $a^2 > b^2$

$$\begin{aligned} \sqrt{4ab - 2i(a^2 - b^2)} &= \sqrt{2(a^2 + b^2)} \left(\cos\left(\frac{1}{2} \operatorname{arctg}\left(\frac{b^2 - a^2}{2ab}\right) - \frac{\pi}{2}\right) + \right. \\ &\quad \left. i \sin\left(\frac{1}{2} \operatorname{arctg}\left(\frac{b^2 - a^2}{2ab}\right) - \frac{\pi}{2}\right) \right); \end{aligned}$$

$$\begin{aligned} \sqrt{4ab - 2i(a^2 - b^2)} &= \sqrt{2(a^2 + b^2)} \left(\cos\left(\frac{1}{2} \operatorname{arctg}\left(\frac{b^2 - a^2}{2ab}\right) + \frac{\pi}{2}\right) + \right. \\ &\quad \left. i \sin\left(\frac{1}{2} \operatorname{arctg}\left(\frac{b^2 - a^2}{2ab}\right) + \frac{\pi}{2}\right) \right). \end{aligned}$$