Question #46172 – Math – Analytic Geometry

Find the vertices, eccentricity, foci and asymptotes of the hyperbola. Also trace it. Under what conditions on the line x^2/8-y^2/4=1 will be tangent to this hyperbola? Explain geometrically.

Solution:

For the hyperbola with equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

We have:

a) the vertices in points (-a, 0) and (a, 0)

b) eccentricity ε is equal $\varepsilon = \frac{c}{a}$ where $c = \sqrt{a^2 + b^2}$

c) the foci in points (-c, 0) and (c, 0), $c = \sqrt{a^2 + b^2}$

d) asymptotes of the hyperbola: $y = \frac{b}{a} * x$, $y = -\frac{b}{a} * x$

e) the line passing through the point of hyperbola (x_0, y_0) and which is tangent to the hyperbola has equation:

$$\frac{x * x_0}{a^2} - \frac{y * y_0}{b^2} = 1$$

So we have hyperbola with equation:

$$\frac{x^2}{8} - \frac{y^2}{4} = 1$$

Therefor hyperbola has:

a) the vertices in points (-2$\sqrt{2}$, 0) and (2$\sqrt{2}$, 0)

b) $c = \sqrt{a^2 + b^2} = \sqrt{8 + 4} = \sqrt{12} = 2\sqrt{3}$, so eccentricity ε is equal $\varepsilon = \frac{c}{a} = \frac{2\sqrt{3}}{2\sqrt{2}} = \sqrt{\frac{3}{2}}$

c) the foci in points (-c2$\sqrt{3}$, 0) and (2$\sqrt{3}$, 0)

d) asymptotes of the hyperbola: $y = \frac{2}{2\sqrt{2}} * x = \frac{\sqrt{2}}{2} * x$, $y = -\frac{\sqrt{2}}{2} * x$.

e) the line passing through the any point hyperbola (x_0, y_0) and which is tangent to the hyperbola has equation:

$$\frac{x * x_0}{8} - \frac{y * y_0}{4} = 1$$

Example:pPoints (4, 2) and (-4, 2) belong to hyperbola. tangent to hyperbola in this points has equations:

$$\frac{x * 4}{8} - \frac{y * 2}{4} = 1 \rightarrow \frac{x}{2} - \frac{y}{2} = 1$$

$$\frac{x * 4}{8} - \frac{y * (-2)}{4} = 1 \rightarrow \frac{x}{2} + \frac{y}{2} = 1$$

Hyperbola was drawn using MAPLE 15:

implicitplot([(1/8)*x^2-(1/4)*y^2 = 1, y = -x/sqrt(2), y = x/sqrt(2), (1/2)*x-(1/2)*y = 1, (1/2)*x+(1/2)*y = 1], x = -20 .. 20, y = -10 .. 10, color = [black, blue, blue, red, yellow], legend = ["graph of a hyperbola", "asymptotes of a hyperbola", "asymptotes of a hyperbola", "tangent to hyperbola in point(4, 2)", "tangent to hyperbola in point(4,-2)"], title = "Graph of hyperbola ", labels = ["x values", "y values"], labeldirections = ["horizontal", "vertical"],

thickness = [2, 1, 1, 1, 1], linestyle = [solid, longdash, longdash, solid, solid], axis = [gridlines = [20, thickness = 1, colour = green, majorlines = 1]])

Graph of hyperbola

graph of a hyperbola — asymptotes of a hyperbola
tangent to hyperbola in point(4, 2) — tangent to hyperbola in point(4,-2)