Answer on Question #46170 – Math – Analytic Geometry

Question. Obtain the equation of the plane Q passing through the line

$$L: \frac{x-2}{2} = -\frac{y+1}{1} = \frac{z-3}{4}$$

and which is perpendicular to the plane $P: x + 2y + z = 4$.

Solution. We have that

- the line L passes through a point $A(2, -1, 3)$ in the direction of the vector $l(2, -1, 4)$,
- the normal vector of the plane P has coordinates $p(1, 2, 1)$.

Let $n(a, b, c)$ be normal vector of the plane Q passing through the line L and perpendicular to Q. Then Q passes through point A, whence its equation has the following form:

$$a(x-2) + b(y+1) + c(z-3) = 0.$$

Notice that n must be perpendicular to both vectors $l(2, -1, 4)$ and $p(1, 2, 1)$, and therefore we can choose n to be the cross product of these vectors:

$$n = l \times p.$$

Thus

$$n = l \times p = (2, -1, 4) \times (1, 2, 1) = \left(\begin{vmatrix} -1 & 4 \\ 2 & 1 \end{vmatrix}, \begin{vmatrix} 4 & 2 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} \right)$$
$$= (-1 \cdot 1 - 2 \cdot 4,\ 4 \cdot 1 - 1 \cdot 2,\ 2 \cdot 2 - 1 \cdot (-1)) = (-9, 2, 5).$$

Hence Q has the following equation:

$$-9(x-2) + 2(y+1) + 5(z-3) = 0$$
$$-9x + 18 + 2y + 2 + 5z - 15 = 0$$
$$-9x + 2y + 5z + 5 = 0.$$

Answer. $-9x + 2y + 5z + 5 = 0$.