Answer on Question #46167 – Math - Analytic Geometry

Problem.

Find the equation of the plane which passes through the line of intersection of the planes 2x+y-2z=6 and 2x+3y+6z=5 and makes equal angles with these planes.

Solution.

Let α is the plane equation of which we should find and (a,b,c) is normal vector of plane α. The plane α makes equal angles with planes $2x+y-2z=6$ and $2x+3y+6z=5$ if and only if normal vectors of α makes equal angles with normal vectors of planes $2x+y-2z=6$ and $2x+3y+6z=5$ or when cosines of these angles are equal. The normal vectors of planes $2x+y-2z=6$ and $2x+3y+6z=5$ are $(2,1,-2)$ and $(2,3,6)$ respectivly. Let φ_1 is angle between (a,b,c) and $(2,1,-2)$ and φ_2 is angle between (a,b,c) and $(2,3,6)$. From inner product formula

$$\cos\varphi_1 = \frac{2a+b-2c}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{2^2+1^2+(-2)^2}},$$

$$\cos\varphi_2 = \frac{2a+3b+6c}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{2^2+3^2+6^2}},$$

So $\frac{2a+b-2c}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{2^2+1^2+(-2)^2}} = \frac{2a+3b+6c}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{2^2+3^2+6^2}}$ ($\cos\varphi_1 = \cos\varphi_2$). Hence

$$14a+7b-14c = 6a+9b+18c$$

or

$$8a-2b-32c=0 \text{ or } b=4a-16c.$$

The line of intersection of the planes $2x+y-2z=6$ and $2x+3y+6z=5$ has equation

$$\begin{cases} 2x+y-2z=6; \\ 2x+3y+6z=5; \\ z=t, \end{cases}$$

where $t\in\mathbb{R}$.

Then $y=-\frac{8t+1}{2}$ and $x=\frac{12t+13}{4}$.

Then the equation of plane α is

$$a\left(x-\frac{12t+13}{4}\right)+b\left(y+\frac{8t+1}{2}\right)+c(z-t)=0$$

or

$$ax+by+cz-\frac{13a}{4}+\frac{b}{2}+(-3a+4b-c)t=0$$

is for all $t\in\mathbb{R}$.

Hence for all t

$$(-3a+4b-c)t=0.$$

Therefore

$$3a-4b+c=0,$$

Hence $3a-4\cdot(4a-16c)+c=0$ (as $b=4a-16c$), $-13a+65c=0$, $a=\frac{65}{13}c$. Then $a=\frac{65}{13}c$ and $b=4c$. The equation of α plane is

$$\frac{65}{13}x+4y+z-\frac{65}{4}+2=0;$$

$$\frac{65}{13}x+4y+z-\frac{57}{4}=0.$$

Answer: $\frac{65}{13}x+4y+z-\frac{57}{4}=0$