Answer on Question #46166 – Math – Analytic Geometry

Question. Find the angle α between the lines

$$L_1: \ x = 1, \qquad z - y = 0$$

and

$$L_2: \ 2x - y = -1, \qquad z = 1.$$

Solution. First we should find the vectors l_1 and l_2 parallel to these lines. The line L_1 is the intersection of two planes $x = 1$ and $z - y = 0$. The normal vectors to these planes are respectively:

$$n_1 = (1, 0, 0), \qquad n_2 = (0, -1, 1).$$

Therefore l_1 is orthogonal to both n_1 and n_2, and so it is parallel to the cross-product of n_1 and n_2. Thus we can assume that

$$l_1 = n_1 \times n_2 = (1, 0, 0) \times (0, -1, 1) = \left(\begin{vmatrix} 0 & 0 \\ -1 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} \right)$$
$$= (0 \cdot 1 - 0 \cdot (-1), \ 0 \cdot 0 - 1 \cdot 1, \ 1 \cdot (-1) - 0 \cdot 0)$$
$$= (0, -1 - 1) .$$

Similarly, the line L_2 is the intersection of two planes $2x - y = -1$ and $z = 1$. The normal vectors to these planes are respectively:

$$p_1 = (2, -1, 0), \qquad p_2 = (0, 0, 1).$$

Therefore l_2 can be taken to be the cross-product of p_1 and p_2:

$$l_2 = p_1 \times p_2 = (2, -1, 0) \times (0, 0, 1) = \left(\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 2 & -1 \\ 0 & 0 \end{vmatrix} \right)$$
$$= (-1 \cdot 1 - 0 \cdot 0, \ 0 \cdot 0 - 1 \cdot 2, \ 2 \cdot 0 - 0 \cdot (-1))$$
$$= (-1, -2, 0).$$

The cosine of the angle α between l_1 and l_2 can be computed by the following formula:

$$\cos\alpha = \frac{(l_1, l_2)}{|l_1| \cdot |l_2|}.$$

We have that the scalar product (l_1, l_2) is equal to the sum of products of the corresponding coordinates:

$$(l_1, l_2) = 0 \cdot (-1) + (-1) \cdot (-2) + (-1) \cdot 0 = 2,$$

and the lengths of these vectors are

$$|l_1| = \sqrt{0^2 + (-1)^2 + (-1)^2} = \sqrt{2},$$
$$|l_2| = \sqrt{(-1)^2 + (-2)^2 + 0^2} = \sqrt{5}.$$

Hence

$$\cos\alpha = \frac{(l_1, l_2)}{|l_1| \cdot |l_2|} = \frac{2}{\sqrt{2} \cdot \sqrt{5}} = \frac{\sqrt{2}}{\sqrt{5}} = \sqrt{2/5} = \sqrt{0.4}.$$

Therefore

$$\alpha = \arccos\sqrt{0.4} \approx \arccos(0.63246) \approx 50.768°.$$

Answer. $\alpha = \arccos\sqrt{0.4} \approx 50.768°$.