Find the path traced by the center of the sphere which touches the lines $\frac{x}{y} = \frac{y}{-1} = \frac{z}{2}$ and $2x = y, y - z = 0$.

Solution

The path traced by the center of the sphere is the locus. The radius of the sphere equals to the distance between the center of the sphere and the points on following lines.

$$r = \frac{x}{y} = \frac{y}{-1} = \frac{z}{2}; s = \frac{x}{1} = \frac{y}{2} = \frac{z}{2}.$$

$$R^2 = (x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = (x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2.$$

$$(x + r^2)^2 + (y + r)^2 + (z - 2r)^2 = (x - s)^2 + (y - 2s)^2 + (z - 2s)^2.$$

$$2r^2 x + 2ry - 4rz + (r^4 + r^2 + 4r^2) = -2sx - 4sy - 4sz + (s^2 + 4s^2 + 4s^2).$$

$$(2r^2 + 2s)x + (2r + 4s)y + (4s - 4r)z + (r^4 + 5r^2 - 9s^2) = 0.$$