Answer on Question #46161 – Math – Statistics and Probability

Question. A box containing 8 light bulbs of which 3 are defective. A bulb is selected from the box and tested. If it is defective, another bulb is selected and tested until a nondefective bulb is chosen. Find the expected number and variance of bulbs chosen.

Solution. Let ξ be the number of bulbs chosen. Then

$$P(\xi = 1) = \frac{C_5^1}{C_8^1} = \frac{5!}{4!} \cdot \frac{7!}{8!} = \frac{5}{8};$$

$$P(\xi = 2) = P(the\ first\ is\ defective, the\ second\ is\ nonedefective) = \frac{C_3^1}{C_8^1} \cdot \frac{C_5^1}{C_7^1} =$$

$$= \frac{3!}{2!} \cdot \frac{7!}{8!} \cdot \frac{5!}{4!} \cdot \frac{6!}{7!} = \frac{3}{8} \cdot \frac{5}{7} = \frac{15}{56};$$

$$P(\xi = 3) = P(the\ first\ and\ the\ second\ are\ defective, the\ third\ is\ nondefective) =$$

$$= \frac{C_3^1}{C_8^1} \cdot \frac{C_2^1}{C_7^1} \cdot \frac{C_5^1}{C_6^1} = \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{5}{6} = \frac{5}{56};$$

$$P(\xi = 4) = P(the\ first, the\ second\ and\ the\ third\ are\ defective) = \frac{C_3^1}{C_8^1} \cdot \frac{C_2^1}{C_7^1} \cdot \frac{C_1^1}{C_6^1} =$$

$$= \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{1}{6} = \frac{1}{56}.$$

Therefore ξ has the next distribution:

Value *k* of ξ	1	2	3	4
Probability of $\xi = k$	$\frac{5}{8}$	$\frac{15}{56}$	$\frac{5}{56}$	$\frac{1}{56}$

The expected number of bulbs chosen is $E\xi = 1 \cdot \frac{5}{8} + 2 \cdot \frac{15}{56} + 3 \cdot \frac{5}{56} + 4 \cdot \frac{1}{56} = \frac{3}{2}$.

Calculate

$$E\xi^2 = 1 \cdot \frac{5}{8} + 4 \cdot \frac{15}{56} + 9 \cdot \frac{5}{56} + 16 \cdot \frac{1}{56} = \frac{39}{14};$$

The variance of bulbs chosen is

$$Var\xi = E\xi^2 - (E\xi)^2 = \frac{39}{14} - \frac{9}{4} = \frac{15}{28}.$$

Answer. $E\xi = \frac{3}{2}, Var\xi = \frac{15}{28}$.