Answer to Question #46053 - Math – Integral Calculus

Question:

Find the integral of

$$\int \sec u \, du.$$

cos(u) + c

tan(u) + c

2sec(u) + c

sin(u) + c

Solution.

Recall that $\sec u = \frac{1}{\cos u}$. Thus, using the equality $\sin^2 u + \cos^2 u = 1$, we get

$$\int \sec u \, du = \int \frac{1}{\cos u} du = \int \frac{\cos u}{\cos^2 u} du = \int \frac{\cos u}{1 - \sin^2 u} du.$$

Now substitute $\sin u = y$, noting that $\cos u \, du = dy$:

$$\int \frac{\cos u}{1 - \sin^2 u} du = \int \frac{1}{1 - y^2} dy = \int \frac{1}{(1-y)(1+y)} dy.$$

Using partial fractions and logarithm properties gives us

$$\int \frac{1}{(1-y)(1+y)} dy$$
$$= \frac{1}{2} \int \left(\frac{1}{1+y} + \frac{1}{1-y} \right) dy = \frac{1}{2} (\ln|1+y| - \ln|1-y|) + C = \frac{1}{2} \ln \left| \frac{y+1}{y-1} \right| + C.$$

Finally, recall that $y = \sin u$ and substitute this into the last expression:

$\frac{1}{2} \ln \left| \frac{y+1}{y-1} \right| + C = \frac{1}{2} \ln \left| \frac{\sin u + 1}{\sin u - 1} \right| + C = \frac{1}{2} \ln \left| \frac{\sin u + 1}{\sin u - 1} \cdot \frac{\sin u + 1}{\sin u + 1} \right| + C = \frac{1}{2} \ln \left| \frac{(\sin u + 1)^2}{1 - \sin^2 u} \right| +$
$C = \frac{1}{2} \ln \left| \left(\frac{\sin u + 1}{\cos u} \right)^2 \right| + C = 2 \cdot \frac{1}{2} \ln \left| \frac{\sin u + 1}{\cos u} \right| + C = \ln \left| \frac{\sin u}{\cos u} + \frac{1}{\cos u} \right| + C = \ln|\tan u + \sec u| + C.$
Correct answer in the statement of question was absent .

Answer.

$$\int \sec u \, du = \ln|\tan u + \sec u| + C.$$