

### Answer on Question #46051 – Math - Integral Calculus

Find the surface area of the band of the sphere generated by revolving the arc of the circle

$$x^2 + y^2 = r^2$$

lying above the interval  $[-a, a]$ ,  $a, r$

$\pi$

$3\pi$

$4\pi ar$

$2\pi ar$

**Solution.**

In the case when  $f(x)$  is positive and has a continuous derivative, the surface area of the surface generated by revolving the curve  $y = f(x)$ ,  $a \leq x \leq b$  about the  $x$  – axis is:

$$S = 2\pi \int_{x_1}^{x_2} y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx.$$

In our case:  $y = \sqrt{r^2 - x^2}$ ,  $x_1 = -a$ ,  $x_2 = a$ ,

$$\frac{dy}{dx} = \frac{1}{2}(r^2 - x^2)^{-\frac{1}{2}}(-2x) = -\frac{x}{\sqrt{r^2 - x^2}}.$$

$$\begin{aligned} \text{So, } S &= 2\pi \int_{-a}^a \sqrt{r^2 - x^2} \sqrt{1 + \frac{x^2}{r^2 - x^2}} dx = 2\pi \int_{-a}^a \sqrt{r^2 - x^2} \frac{r}{\sqrt{r^2 - x^2}} dx = \\ &= 2\pi r \int_{-a}^a dx = 2\pi r x \Big|_{x=-a}^{x=a} = 2\pi ar + 2\pi ar = 4\pi ar. \end{aligned}$$

**Right answer is #3.**