

Answer on Question #45961 – Math – Integral Calculus

Find the surface area of the band of the sphere generated by revolving the arc of the circle $x^2 + y^2 = r^2$ lying above the interval $[-a, a]$, a, r

Solution.

In the case when $f(x)$ is positive and has a continuous derivative, the surface area of the surface generated by revolving the curve $y = f(x)$, $x_1 \leq x \leq x_2$ about the x – axis is

$$S = 2\pi \int_{x_1}^{x_2} y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx.$$

In our case: $y = \sqrt{r^2 - x^2}$, $x_1 = -a$, $x_2 = a$,

$$\frac{dy}{dx} = \frac{1}{2}(r^2 - x^2)^{-\frac{1}{2}}(-2x) = -\frac{x}{\sqrt{r^2 - x^2}}.$$

Thus,

$$\begin{aligned} S &= 2\pi \int_{-a}^a \sqrt{r^2 - x^2} \sqrt{1 + \frac{x^2}{r^2 - x^2}} dx = 2\pi \int_{-a}^a \sqrt{r^2 - x^2} \frac{r}{\sqrt{r^2 - x^2}} dx = \\ &= 2\pi r \int_{-a}^a dx = 2\pi r x \Big|_{x=-a}^{x=a} = 2\pi ar + 2\pi ar = 4\pi ar. \end{aligned}$$