

Answer on Question #45960 – Math – Integral Calculus

Question.

Find the area of the region bounded by the graphs of \sqrt{x} and $y = -x - 1$ between $x = 1$ and $x = 4$.

Solution.

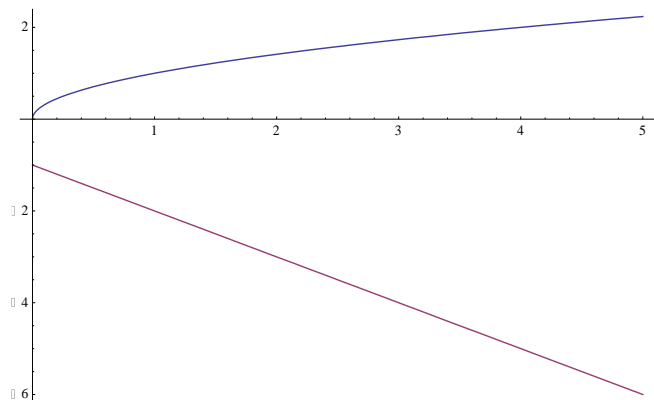


Fig.1. Our functions.

As we know, the area under the non-negative function is defined as integral of non-negative function. But in our case, we must calculate the sum of two integrals with two functions. Because the integral gives only the area between the curve and the x-axes.

Function \sqrt{x} is non-negative for $1 \leq x \leq 4$, function $(-x - 1)$ is negative for $1 \leq x \leq 4$. To evaluate the area bounded by the graph of $y = -x - 1$ and x-axis between $x = 1$ and $x = 4$, we take expression $(-x - 1)$ with the opposite sign in the definite integral.

So, let calculate the area between the functions:

$$\begin{aligned} A &= \int_1^4 \sqrt{x} \, dx - \int_1^4 (-x - 1) \, dx = \frac{2}{3} x^{\frac{3}{2}} \Big|_1^4 + \int_1^4 (x + 1) \, dx = \frac{2}{3} (8 - 1) + \left(\frac{x^2}{2} + x \right) \Big|_1^4 = \\ &= \frac{14}{3} + (8 + 4 - \frac{1}{2} - 1) = \frac{14}{3} + \frac{21}{2} = \frac{28 + 63}{6} = \frac{91}{6} \end{aligned}$$

Answer.

$$A = \frac{91}{6}$$