

Answer to Question #45959 - Math – Integral Calculus

Question:

Find the integral of

$$\int \sec u \, du.$$

Solution.

Recall that $\sec u = \frac{1}{\cos u}$. Thus, using the equality $\sin^2 u + \cos^2 u = 1$, we get

$$\int \sec u \, du = \int \frac{1}{\cos u} \, du = \int \frac{\cos u}{\cos^2 u} \, du = \int \frac{\cos u}{1 - \sin^2 u} \, du.$$

Now substitute $\sin u = y$, noting that $\cos u \, du = dy$:

$$\int \frac{\cos u}{1 - \sin^2 u} \, du = \int \frac{1}{1 - y^2} \, dy = \int \frac{1}{(1 - y)(1 + y)} \, dy.$$

Using partial fractions and logarithm properties gives us

$$\begin{aligned} \int \frac{1}{(1 - y)(1 + y)} \, dy \\ = \frac{1}{2} \int \left(\frac{1}{1 + y} + \frac{1}{1 - y} \right) \, dy = \frac{1}{2} (\ln|1 + y| - \ln|1 - y|) + C = \frac{1}{2} \ln \left| \frac{y + 1}{y - 1} \right| + C. \end{aligned}$$

Finally, recall that $y = \sin u$ and substitute this into the last expression:

$$\begin{aligned} \frac{1}{2} \ln \left| \frac{y + 1}{y - 1} \right| + C &= \frac{1}{2} \ln \left| \frac{\sin u + 1}{\sin u - 1} \right| + C = \frac{1}{2} \ln \left| \frac{\sin u + 1}{\sin u - 1} \cdot \frac{\sin u + 1}{\sin u + 1} \right| + C \\ &= \frac{1}{2} \ln \left| \frac{(\sin u + 1)^2}{1 - \sin^2 u} \right| + C = \frac{1}{2} \ln \left| \left(\frac{\sin u + 1}{\cos u} \right)^2 \right| + C = 2 \cdot \frac{1}{2} \ln \left| \frac{\sin u + 1}{\cos u} \right| + C \\ &= \ln \left| \frac{\sin u}{\cos u} + \frac{1}{\cos u} \right| + C = \ln |\tan u + \sec u| + C \end{aligned}$$

Answer.

$$\int \sec u \, du = \ln |\tan u + \sec u| + C.$$