Answer on Question #45950 – Math – Statistics and Probability

Problem.

A ship is fitted with three engines A, B and C. These three engines function independently of each other with probabilities ½ , ¼, ¼ respectively. For ship to be operational at least two of its engines must function. Let E denote the event that the ship is operational and E1,E2 and E3 denote the events that engines A,B and C are functioning. Then validate the following.

(a). P(E1C / E) = 3/16

(b) P(E / E2) = 5/16

(c) P(Exactly two engines of the ship are functioning / E) = 7/8.

Solution:

The ship is operational if all three engines are functioning or when exactly one of the three engines is not functioning and the other two engines are functioning. This four events are mutually independent, so

$$P(E) = P(E_1E_2E_2) + P(E_1^cE_2E_3) + P(E_1E_2^cE_3) + P(E_1E_2E_3^c).$$

All three events in the triples $E_1E_2E_2$ (all three engines are functioning), $E_1^cE_2E_3$ (only 2 and 3 engines are functioning), $E_1E_2^cE_3$ (only 1 and 3 engines are functioning), $E_1E_2E_3^c$ (only 1 and 2 engines are functioning) are mutually independent, so

$$P(E) = \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4} + \left(1-\frac{1}{2}\right)\cdot\frac{1}{4}\cdot\frac{1}{4} + \frac{1}{2}\cdot\left(1-\frac{1}{4}\right)\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{1}{4}\cdot\left(1-\frac{1}{4}\right)$$
$$= \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{4} = \frac{1}{32} + \frac{1}{32} + \frac{3}{32} + \frac{3}{32} = \frac{1}{4}.$$

Hence

(a)

$$P(E_1^c|E) = \frac{P(E_1^c \cap E)}{P(E)} = \frac{P(E_1^cE_2E_3)}{P(E)} = \frac{\frac{3}{32}}{\frac{1}{4}} = \frac{3}{8}.$$

(b)

$$P(E|E_2) = \frac{P(E \cap E_2)}{P(E_2)} = \frac{P(E_1E_2E_2) + P(E_1^cE_2E_3) + P(E_1E_2E_3^c)}{P(E_2)} = \frac{\frac{1}{32} + \frac{1}{32} + \frac{3}{32}}{\frac{1}{4}} = \frac{\frac{5}{32}}{\frac{1}{4}} = \frac{5}{8}.$$

(c)

$$P(\text{Exactly two engines of the ship are functioning}|E)$$
$$= \frac{P(\text{Exactly two engines of the ship are functioning} \cap E)}{P(E)}$$
$$= \frac{P(E_1^cE_2E_3) + P(E_1E_2^cE_3) + P(E_1E_2E_3^c)}{P(E)} = \frac{\frac{1}{32} + \frac{3}{32} + \frac{3}{32}}{\frac{1}{4}} = \frac{7}{8}.$$

Hence (a) and (b) are false and (c) is true.

Answer: (a) and (b) are false and (c) is true