Answer on Question #45894 – Math – Statistics and Probability

Question.

Let X be a continuous random variable with the probability distribution defined by

$p(x) = \begin{cases} kx^2, 0 \le x \le 3 \\ 0 \; elsewhere \end{cases}$. Evaluate k and find

(i) $P(1 \le X < 2)$, **(ii)** $P(X \le 1)$, **(iii)** $P(X > 1)$.

Also find the mean and variance of the distribution.

Solution.

According to the question, $p(x)$ is a probability distribution function $\Rightarrow \int_{-\infty}^{+\infty} p(x)dx = 1 \Rightarrow$

$$0 + \int_0^3 kx^2 dx = 1 \Rightarrow$$

$\Rightarrow \frac{k}{3}x^3|_0^3 = 9k = 1 \Rightarrow k = \frac{1}{9}$.

(i) $P(1 \le X < 2) = \int_1^2 \frac{1}{9}x^2 dx = \frac{x^3}{27}|_1^2 = \frac{7}{27}$;

(ii) $P(X \le 1) = \int_0^1 \frac{1}{9}x^2 dx = \frac{x^3}{27}|_0^1 = \frac{1}{27}$;

(iii) $P(X > 1) = \int_1^3 \frac{1}{9}x^2 dx = \frac{x^3}{27}|_1^3 = \frac{26}{27}$

The mean: $EX = \int_{-\infty}^{+\infty} xp(x)dx = \frac{1}{9}\int_0^3 x^3 dx = \frac{1}{36}x^4|_0^3 = \frac{81}{36} = \frac{9}{4}$.

$EX^2 = \int_{-\infty}^{+\infty} x^2 p(x)dx = \frac{1}{9}\int_0^3 x^4 dx = \frac{1}{45}x^5|_0^3 = \frac{243}{45} = \frac{27}{5}$.

The variance: $VarX = EX^2 - (EX)^2 = \frac{27}{5} - \frac{81}{16} = \frac{27}{80}$.

Answer. $k = \frac{1}{9}$;

(i) $\frac{7}{27}$;

(ii) $\frac{1}{27}$;

(iii) $\frac{26}{27}$;

$EX = \frac{9}{4}$; $VarX = \frac{27}{80}$.