

Answer on Question #45893 – Math – Statistics and Probability

Problem.

Let X , and Y be two independent random variables such that $E(X)=2$, $\text{Var}(X)=4$, $E(Y)=7$, and $\text{Var}(Y)=1$.
Find:

- 1). $E(3X+7)$ and $\text{Var}(3X+7)$
- 2). $E(5X+2Y-2)$ and $\text{Var}(5X+2Y-2)$.

(b) If two random variables have the joint density
Find the probability that

Remark:

The part of b question is missed, so this is the answer to a question.

Solution:

1) The expectation operator is linear, so

$$E(3X + 7) = E(3X) + E(7) = 3E(X) + E(7) = 6 + 7 = 13.$$

$$\text{Var}(X) = E(X^2) - (E(X))^2 = 4, \text{ so } E(X^2) = \text{Var}(X) + (E(X))^2 = 8.$$

$$\begin{aligned}\text{Var}(3X + 7) &= E((3X + 7)^2) - (E(3X + 7))^2 = E((3X + 7)^2) - 169 = \\ &= E(9X^2 + 42X + 49) - 169 = E(9X^2) + E(42X) + E(49) - 169 = \\ &= 9E(X^2) + 42E(X) + E(49) - 169 = \\ &= 9 \cdot 8 + 42 \cdot 2 + 49 - 169 = 36, \text{ by the definition of the variance.}\end{aligned}$$

Other method uses properties of variances:

$$\text{Var}(3X + 7) = \text{Var}(3X) = 3^2\text{Var}(X) = 9\text{Var}(X) = 9 \cdot 4 = 36$$

2) The expectation operator is linear, so

$$E(5X + 2Y - 2) = E(5X) + E(2Y) + E(-2) = 5E(X) + 2E(Y) - 2 = 10 + 14 - 2 = 22.$$

$$\text{Var}(X) = E(X^2) - (E(X))^2 = 4 \text{ and } \text{Var}(Y) = E(Y^2) - (E(Y))^2 = 1, \text{ so}$$

$$E(X^2) = \text{Var}(X) + (E(X))^2 = 8 \text{ and } E(Y^2) = \text{Var}(Y) + (E(Y))^2 = 50.$$

$E(X)E(Y) = E(XY)$, as X and Y are independent.

$$\begin{aligned}\text{Var}(5X + 2Y - 2) &= E((5X + 2Y - 2)^2) - (E(5X + 2Y - 2))^2 = E((5X + 2Y - 2)^2) - 484 = \\ &= E(25X^2 + 4Y^2 + 4 + 20XY - 20X - 8Y) - 484 = \\ &= E(25X^2) + E(4Y^2) + E(4) + E(20XY) + E(-20X) + E(-8Y) - 484 = \\ &= 25E(X^2) + 4E(Y^2) + E(4) + 20E(X)E(Y) - 20E(X) - 8E(Y) - 484 = 104\end{aligned}$$

by the definition of the variance.

Other method uses properties of variances:

$$\begin{aligned}\text{Var}(5X + 2Y - 2) &= \text{Var}(5X + 2Y) = |X \text{ and } Y \text{ are independent variables}| = \\ &= \text{Var}(5X) + \text{Var}(2Y) = 5^2\text{Var}(X) + 2^2\text{Var}(Y) = 25\text{Var}(X) + 4\text{Var}(Y) = \\ &= 25 \cdot 4 + 4 \cdot 1 = 104\end{aligned}$$

Answer:

- 1) $E(3X + 7) = 13$, $\text{Var}(3X + 7) = 36$.
- 2) $E(5X + 2Y - 2) = 22$, $\text{Var}(5X + 2Y - 2) = 104$.