

Answer on Question #45892 – Math – Statistics and Probability

Question.

(a) During a laboratory experiment the average number of radioactive particles passing through a counter in 1 millisecond is 4. What is a probability that 6 particles enter the counter in a given millisecond?

(b) The height of adult women in the United States is normally distributed with mean 64.5 inches and S.D 2.4 inches. Find the probability that a randomly chosen women is **a)** less than 63 inches tall **b)** less than 70 inches tall **c)** between 63 and 70 inches tall. **d)** Alice is 72 inches tall. What percentage of women is shorter than Alice?

Solution.

(a) Assume that the particle stream is the easiest. Then the number ξ of radioactive particles passing through a counter in 1 millisecond has the Poisson's distribution, i. e.

$$P(\xi = k) = \frac{\lambda^k}{k!} e^{-\lambda} \text{ where } k = 0, 1, 2, \dots, \text{ and by the hypothesis } \lambda = 4. \text{ So,}$$
$$P(\xi = 6) = \frac{4^6}{6!} e^{-4} \approx 0.104.$$

(b) Let ξ be the height of adult women in the United States. Then $\eta = \frac{\xi - 64.5}{2.4}$ has a standard normal distribution, i. e. $P(\eta < x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{u^2}{2}} du$. The tabulated function of Laplace has the form $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{u^2}{2}} du$.

$$\text{a) } P(\xi < 63) = P\left(\frac{\xi - 64.5}{2.4} < \frac{63 - 64.5}{2.4}\right) = P\left(\frac{\xi - 64.5}{2.4} < -0.625\right) = 0.5 - \Phi(0.625) =$$
$$= 0.5 - 0.23565 = 0.26435.$$

$$\text{b) } P(\xi < 70) = P\left(\frac{\xi - 64.5}{2.4} < \frac{70 - 64.5}{2.4}\right) = P\left(\frac{\xi - 64.5}{2.4} < 2.29\right) = 0.5 + \Phi(2.29) = 0.5 + 0.48928 =$$
$$= 0.98928.$$

$$\text{c) } P(63 < \xi < 70) = P\left(\frac{63 - 64.5}{2.4} < \frac{\xi - 64.5}{2.4} < \frac{70 - 64.5}{2.4}\right) = P\left(-0.625 < \frac{\xi - 64.5}{2.4} < 2.29\right) =$$
$$= \Phi(0.625) + \Phi(2.29) = 0.23565 + 0.48928 = 0.72493.$$

$$\text{d) } P(\xi < 72) = P\left(\frac{\xi - 64.5}{2.4} < \frac{72 - 64.5}{2.4}\right) = P\left(\frac{\xi - 64.5}{2.4} < 3.125\right) = 0.5 + \Phi(3.125) =$$
$$= 0.5 + 0.49903 = 0.99903 = 99.903\%.$$

Answer. (a) 0.104

(b):

a) 0.26435

b) 0.98928

c) 0.72493

d) 99.903%