

Answer on Question #45890 – Math – Statistics and Probability

Question.

(a) The chances that an academician, a business man and a politician becoming Vice Chancellor of a university are 0.5, 0.3 and 0.2 respectively. The probability that research work will be promoted in the university by these 3 gentlemen are respectively are 0.8, 0.6 and 0.4. It is found research work has been promoted by the university. What is the chance that an academician has become the VC?

(b) A can hit a target 3 times in 5 shots; B can hit 2 times in 5 shots; C can hit 3 times in 4 shots. They fire a volley. What is the probability that two shots hit?

Solution.

(a) Let event H_1 – an academician became Vice Chancellor; H_2 – a business man became Vice Chancellor; H_3 – a politician became Vice Chancellor. Let event A – research work is promoted in the university. Then $P(H_1) = 0.5, P(H_2) = 0.3, P(H_3) = 0.2, P\left(\frac{A}{H_1}\right) = 0.8,$

$P(A/H_2) = 0.6, P(A/H_3) = 0.4$. To find the probability that an academician has become the VC we shall use the Bayes formula:

$$P(H_1/A) = \frac{P(A/H_1) \cdot P(H_1)}{P(A/H_1) \cdot P(H_1) + P(A/H_2) \cdot P(H_2) + P(A/H_3) \cdot P(H_3)} = \frac{0.8 \cdot 0.5}{0.8 \cdot 0.5 + 0.6 \cdot 0.3 + 0.4 \cdot 0.2} = \frac{20}{33}.$$

(b) Let event A = "A hit a target", B = "B hit a target", C = "C hit a target".

$$\text{Then } P(A) = \frac{3}{5} = 0.6,$$

$$P(B) = \frac{2}{5} = 0.4, P(C) = \frac{3}{4} = 0.75. \text{ Assume that events } A, B, C \text{ are independent.}$$

$$\begin{aligned} P(\text{two shots hit}) &= P(A \cap B \cap \bar{C}) + P(A \cap \bar{B} \cap C) + P(\bar{A} \cap B \cap C) = \\ &= P(A) \cdot P(B) \cdot P(\bar{C}) + P(A) \cdot P(\bar{B}) \cdot P(C) + P(\bar{A}) \cdot P(B) \cdot P(C) = \\ &= P(A) \cdot P(B) \cdot (1 - P(C)) + P(A) \cdot (1 - P(B)) \cdot P(C) + (1 - P(A)) \cdot P(B) \cdot P(C) = \\ &= 0.6 \cdot 0.4 \cdot 0.25 + 0.6 \cdot 0.6 \cdot 0.75 + 0.4 \cdot 0.4 \cdot 0.75 = 0.45. \end{aligned}$$

Answer.

(a) $\frac{20}{33}$

(b) 0.45