Answer on Question #45889 – Math – Statistics and Probability

Let X denote the temperature ( ) and let Y denote the time in minutes that it takes for the diesel engine on an automobile to get ready to start. Assume that the joint density function is $f_{XY}(x,y) = c\,(4x+2y+1)$.

$0 \le x \le 40,\ 0 \le y \le 2$

i) Find c, **ii)** Find the marginal densities for X and Y, **iii)** Find the probability that on a randomly selected day the air air temperature will exceed **iv)** Are X and Y independent?

Solution

i) The integral of the joint distribution over its domain must be 1. So,

$\int_0^{40} dx \int_0^2 dy\, c(4x + 2y + 1) = \int_0^2 dy\, c(2x^2 + 2xy + x)(x = 40) - \int_0^2 dy\, c(2x^2 + 2xy + x)(x = 0) = \int_0^2 dy\, c(80\,y + 3240) = 40c[y^2 + 81y](y = 2) - 40c[y^2 + 81y](y = 0) = 40c\,[4 + 162] = 6640c = 1$

$$c = \frac{1}{6640}.$$

ii) The marginal distribution Fax for x is

$$f_x(x) = \int_0^2 f_{XY}(x,y)dy = \frac{1}{6640}\int_0^2 dy\,(4x + 2y + 1) =$$
$$= \frac{1}{6640}[4xy + y^2 + y](y = 2) - \frac{1}{6640}[4xy + y^2 + y](y = 0)$$
$$= \frac{1}{6640}(8x + 6).$$

For y, we have

$$f_y(y) = \int_0^{40} f_{XY}(x,y)dx = \frac{1}{6640}\int_0^{40} dx\,(4x + 2y + 1) =$$
$$= \frac{1}{6640}[2x^2 + 2yx + x]\,(x = 40) - \frac{1}{6640}[2x^2 + 2yx + x](x = 0)$$
$$= \frac{1}{6640}(80y + 3240).$$

iii) It's the probability that $X > X_0$ and $Y > Y_0$. So,

$$P(X > X_0, Y > Y_0) = \frac{1}{6640}\int_{X_0}^{40} dx \int_{Y_0}^{2} dy\,(4x + 2y + 1)$$
$$= \frac{4}{6640}\int_{X_0}^{40} xdx \int_{Y_0}^{2} dy + \frac{2}{6640}\int_{X_0}^{40} dx \int_{Y_0}^{2} ydy + \frac{1}{6640}\int_{X_0}^{40} dx \int_{Y_0}^{2} dy =$$
$$= \frac{4}{6640}\frac{x^2}{2}\Big|_{x=X_0}^{40} y|_{y=Y_0}^{2} + \frac{2}{6640}x|_{x=X_0}^{40}\frac{y^2}{2}\Big|_{y=Y_0}^{2} + \frac{1}{6640}x|_{x=X_0}^{40} y|_{y=Y_0}^{2} =$$
$$= \frac{1}{3320}(40^2 - X_0^2)(2 - Y_0) + \frac{1}{6640}(40 - X_0)(2^2 - Y_0^2) +$$

$+\frac{1}{6640}(40 - X_0)(2 - Y_0)$ over the rectangle

$\{X_0 < x \le 40, Y_0 \le y \le 2\}$.

iv) No, because their joint distribution is not the product of their marginal distributions.