

Answer on Question #45888 – Math – Statistics and Probability

Problem.

A ship is fitted with three engines A, B and C. These three engines function independently of each other with probabilities $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{4}$ respectively. For ship to be operational at least two of its engines must function. Let E denote the event that the ship is operational and E_1, E_2 and E_3 denote the events that engines A, B and C are functioning. Then validate the following.

(a). $P(E_1 C / E) = 3/16$

(b) $P(E / E_2) = 5/16$

(c) $P(\text{ Exactly two engines of the ship are functioning } / E) = 7/8.$

Solution:

The ship is operational if all three engines are functioning or when exactly one of the three engines is not functioning and the other two engines are functioning. This four events are mutually independent, so

$$P(E) = P(E_1 E_2 E_3) + P(E_1^c E_2 E_3) + P(E_1 E_2^c E_3) + P(E_1 E_2 E_3^c).$$

All three events in the triples $E_1 E_2 E_3$ (all three engines are functioning), $E_1^c E_2 E_3$ (only 2 and 3 engines are functioning), $E_1 E_2^c E_3$ (only 1 and 3 engines are functioning), $E_1 E_2 E_3^c$ (only 1 and 2 engines are functioning) are mutually independent, so

$$\begin{aligned} P(E) &= \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{4} + \left(1 - \frac{1}{2}\right) \cdot \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{2} \cdot \left(1 - \frac{1}{4}\right) \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{4} \cdot \left(1 - \frac{1}{4}\right) \\ &= \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{3}{4} = \frac{1}{32} + \frac{1}{32} + \frac{3}{32} + \frac{3}{32} = \frac{1}{4}. \end{aligned}$$

Hence

(a)

$$P(E_1^c | E) = \frac{P(E_1^c \cap E)}{P(E)} = \frac{P(E_1^c E_2 E_3)}{P(E)} = \frac{\frac{3}{32}}{\frac{1}{4}} = \frac{3}{8}.$$

(b)

$$P(E|E_2) = \frac{P(E \cap E_2)}{P(E_2)} = \frac{P(E_1 E_2 E_3) + P(E_1^c E_2 E_3) + P(E_1 E_2 E_3^c)}{P(E_2)} = \frac{\frac{1}{32} + \frac{1}{32} + \frac{3}{32}}{\frac{1}{4}} = \frac{\frac{5}{32}}{\frac{1}{4}} = \frac{5}{8}.$$

(c)

$$\begin{aligned} &P(\text{Exactly two engines of the ship are functioning} | E) \\ &= \frac{P(\text{Exactly two engines of the ship are functioning} \cap E)}{P(E)} \\ &= \frac{P(E_1^c E_2 E_3) + P(E_1 E_2^c E_3) + P(E_1 E_2 E_3^c)}{P(E)} = \frac{\frac{1}{32} + \frac{3}{32} + \frac{3}{32}}{\frac{1}{4}} = \frac{7}{8}. \end{aligned}$$

Hence (a) and (b) are false and (c) is true.

Answer: (a) and (b) are false and (c) is true