(b) The trouble shooting capability of an IC chip in a circuit is a random variable X whose distribution function is given by

$$F(x) = \begin{cases} 0, \ x \leq 3 \\ 1 - \dfrac{9}{x^2}, x > 3 \end{cases}$$

where x denote the number of years. Find the probability that the IC chip will work properly (i) less than 8 years (ii) beyond 8 years (iii) between 5 to 7 years (iv) anywhere from 2 to 5 years

Solution

This is cumulative distribution function (not the probability distribution function). Because in fact, $\int_3^\infty F(x)dx = \infty$ (it is not equal to 1). But it must be 1 if $F(x)$ is the probability distribution function.

(i)   The probability that the IC chip will work properly less than 8 years is
$$F(8) = 1 - \frac{9}{8^2} = 0.859375.$$

(ii)   The probability that the IC chip will work properly beyond 8 years is
$$1 - F(8) = 1 - 0.859375 = 0.140625.$$

(iii)   The probability that the IC chip will work properly between 5 to 7 years 8 is
$$F(7) - F(5) = \frac{9}{5^2} - \frac{9}{7^2} = 0.1763.$$

(iv)   The probability that the IC chip will work properly anywhere from 2 to 5 years is
$$F(5) - F(3) = \frac{9}{3^2} - \frac{9}{5^2} = 0.64.$$