Answer on Question #45837 – Math – Statistics and Probability

A city council fire brigade officer claims that it takes, on average 10.9 minutes to reach its destination in emergency calls. To check on this claim, the agency which licenses fire brigade service providers had them timed on 50 emergency calls, getting a mean of 10.3 minutes with a standard deviation of 0.8 minutes. At the level of significance of 0.05, does this constitute evidence that the figure claimed is too high?

Solution:

In given problem we need to construct the relevant hypothesis test. Hypothesis testing is the use of statistics to determine the probability that a given hypothesis is true. The usual process of hypothesis testing consists of four steps.

1. Formulate the null hypothesis H_0 (commonly, that the observations are the result of pure chance) and the alternative hypothesis H_a (commonly, that the observations show a real effect combined with a component of chance variation).

2. Identify a test statistic that can be used to assess the truth of the null hypothesis.

3. Compute the P-value, which is the probability that a test statistic at least as significant as the one observed would be obtained assuming that the null hypothesis were true. The smaller the P-value, the stronger the evidence against the null hypothesis.

4. Compare the p-value to an acceptable significance value α alpha (sometimes called an alpha value). If $p \leq \alpha$ alpha, that the observed effect is statistically significant, the null hypothesis is ruled out, and the alternative hypothesis is valid.

We have the following data according to the condition of the task.

$\bar{x} = 10.3, \mu_0 = 10.9, \sigma = 0.8, n = 50$, level of significance is equal to 0.05.

First, we write down the null and alternative hypotheses :

H_0: $\mu = 10.9$

H_1: $\mu < 10.9$

Now we calculate the test statistic. We compute the z score:

$Z = \frac{\mu - \mu_0}{\sigma/\sqrt{n}} = \frac{10.3 - 10.9}{0.8/\sqrt{50}} = \frac{-0.6}{0.11314} \approx -5.303.$

p-value=$\Phi(Z) = \Phi(-5.303) < 0.05.$

Critical value is $-Z_{0.05} = -1.645$. Rejection rule: $we\ reject\ H_0\ if\ Z \leq -Z_{0.05}.$ So we reject H_0 and accept H_1. Therefore, we can conclude that less time 10.9 minutes is needed to reach its destination in emergency calls .