Answer on Question #45787 – Math – Calculus

The price p (in dollars) and demand x for a product are related by

$$x^2 + 2xp + 25p^2 = 74000$$

If the demand is decreasing at a rate of 4 units per month when the demand is 100, find the rate change of the price.

Solution:

In our task we have the following data, p (price) and x (demand) are functions of t (time). So we can noted their rates of change over time $\frac{dp}{dt}$ and $\frac{dx}{dt}$. We know that according to the condition of the task $\frac{dx}{dt} = -4$

We have deal with Implicit differentiation. Firstly we rewrite given equation.

$$x^2 + 2xp + 25p^2 - 74000 = 0$$

Differentiating our equation with respect to t:

$$2x\left(\frac{dx}{dt}\right) + 2p\left(\frac{dx}{dt}\right) + 2x\left(\frac{dp}{dt}\right) + 50p\left(\frac{dp}{dt}\right) = 0$$

We rearrange this to collect all terms involving $\frac{dx}{dt}$ and $\frac{dp}{dt}$ together. So we obtained the following.

$$(2x + 2p)\left(\frac{dx}{dt}\right) + (2x + 50p)\left(\frac{dp}{dt}\right) = 0$$

Then we can rewrite our equation in another manner.

$$(2x + 50p)\left(\frac{dp}{dt}\right) = -(2x + 2p)\left(\frac{dx}{dt}\right)$$

From the obtained equation we can express the value of $\left(\frac{dp}{dt}\right)$.

$$\left(\frac{dp}{dt}\right) = -\frac{(2x + 2p)}{(2x + 50p)}\left(\frac{dx}{dt}\right)$$

According to the condition of the task we know that demand is decreasing at a rate of 4 units per month, this mean the value of $\left(\frac{dx}{dt}\right) = -4$ when the demand is equal to 100. So we need to find the value of price when x =100. Substitute the indicate value into the original equation.

$$(100)^2 + 2(100)p + 25p^2 = 74000$$

Add -74000 to both sides of the equation and multiply terms in the parenthesis.

$$10\,000 + 200p + 25p^2 - 74000 = 0$$

Simplify by combining like terms.

$$25\mathrm{p}^{2}+200\mathrm{p}-64000=0$$

So, we obtained the quadratic equation. Firstly we simplify the equation by dividing all terms by 25.

$$\mathrm{p}^{2}+8\mathrm{p}-2560=0$$

We can solve the given equation by applying the quadratic formula. We have a=1, b=8 and c=-2560. Plug in the values for a, b, and c into the quadratic formula.

$$\mathrm{p}_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

We obtained the following result.

$$p_{1,2}=\frac{-8\pm\sqrt{64+10240}}{2}$$

Simplify expression under the square root.

$$p_{1,2}=\frac{-8\pm\sqrt{10304}}{2}$$

Now we can note separately our roots.

$$p_1=\frac{-8+\sqrt{10304}}{2}=\frac{93.509}{2}\approx 46.7545$$

$$p_2=\frac{-8-\sqrt{10304}}{2}=\frac{-109.509}{2}\approx -54.7545$$

According to the statement of the problem we have observed a decrease in demand, this means an increase in price, so the value of price cannot be negative. Thus we choose the value of price, which is equal to $p_1 \approx 46.75$.

No we can find the the rate change of the price $\left(\frac{dp}{dt}\right)$. Point values are substitute into equation.

$$\left(\frac{dp}{dt}\right)=-\frac{(2(100)+2(46.75))}{(2(100)+50(46.75))}\cdot(-4)$$

Simplify by multiplying terms in the parenthesis.

$$\left(\frac{dp}{dt}\right)=-\frac{(293.5)}{(2537.5)}\cdot(-4)=\frac{1174}{2537.5}=0.4627$$

If the demand is decreasing at a rate of 4 units per month when the demand is 100 then the price increases at the rate of 0.46 dollars/month.

Answer: Rate of change of the price is equal to $\frac{dp}{dt} = 0.46\,\frac{\text{dollars}}{\text{month}}$ or $46\,\frac{\text{cents}}{\text{month}}$.