Answer on Question #45641 – Math – Calculus

A closed box whose length is twice width is to have a surface of 192 square units. find the dimensions of the box when the volume is maximum

Solution:

It's a closed box, so it has a top.

Let x be the width of the box. Then its length is $2x$. And the top and the bottom of the box are each a rectangle with width x and length $2x$, meaning the area is $2x^2$. So the surface area of the four sides (left, right front, back) must be $192 - 4x^2$. The distance around the outside (summed length of the four sides, which is the same as the perimeter as viewed from above, is

$$2x + x + 2x + x = 6x$$

dividing surface area 192 - $4x^2$ by the length round the outside, $6x$, gives the height of the box.

This will be:

$$\frac{192}{6x} - \frac{4x^2}{6x} = \frac{32}{x} - \frac{2x}{3}$$

Now the volume of the box is width times length times height

$$x(2x)\left(\frac{32}{x} - \frac{2x}{3}\right) = (2x)\left(32 - \frac{2x^2}{3}\right) = 64x - \frac{4x^3}{3}.$$

Now we need to maximise this quantity, so we must differentiate this (to get $64 - 4x^2$. This must be set equal to zero, i.e $64 - 4x^2 = 0$, so $4x^2 = 64$, and dividing through by 4 results in

$$x^2 = 16,$$

so x = 4 or -4.

But it's a width, so x cannot be negative and hence x = 4.

So the width is x, which is 4.

The length is $2x = 2 \cdot 4 = 8$.

The height is $\frac{32}{x} - \frac{2x}{3} = \frac{32}{4} - 2 \cdot \frac{4}{3} = 8 - \frac{8}{3} = 5\frac{1}{3}$.

All measurements are in inches, since that's the units we were using.

Answer: width = 4; length = 8; height = $5\frac{1}{3}$