Answer on Question #45607 – Math - Statistics and Probability

(a) A stamping machine produces 'can tops' whose diameters are normally distributed with a standard deviation of 0.02 inch. At what nominal mean diameter should the machine be set, so that no more than 9 % of the 'can tops' produced have diameters exceeding 3.5 inches?

(b) Let A and B be independent events with $P\ (A) = \frac{1}{4}$ and $P\ (A\ U\ B)\ =\ 2P\ (B)\ -\ P\ (A)$.

Find (a). P (B); (b).P (A|B); and (c). P (Bc|A).

Solution

(a) Let X be the diameter of a can top produced by the machine, then X is assumed a normal distribution with to - be-determined mean μ and standard deviation 0.01. From the question we need to consider $P(X > 3.5) < 0.09$.

So we solve

$$0.09 > P(X > 3.5) = P\left(Z > \frac{3.5 - \mu}{0.02}\right).$$

From tables on the standard normal distribution, we have $\frac{3.5-\mu}{0.02} > 1.34$, and therefore it should be set

$\mu < 3.473$ inch.

(b)

(a). The probability of the union of A and B

$$P(A\ U\ B) = P(A) + P(B) - P(A\ \cap\ B).$$

Since A and B are independent:

$$P(A \cap B) = P(A) \cdot P(B).$$

And we have:

$$P(A\ U\ B) = P(A) + P(B) - P(A) \cdot P(B) = 2P(B) - P(A).$$

$$\frac{1}{4} + P(B) - \frac{1}{4} \cdot P(B) = 2P(B) - \frac{1}{4}.$$

$$P(B) = \frac{2}{5}.$$

(b).

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \cdot P(B)}{P(B)} = P(A) = \frac{1}{4}.$$

(c). Since A and B are independent A and B^c are also independent. That's why

$$P(B^c|A) = \frac{P(B^c \cap A)}{P(A)} = \frac{P(B^c) \cdot P(A)}{P(A)} = P(B^c) = 1 - P(B) = 1 - \frac{2}{5} = \frac{3}{5}.$$