

Answer on Question #45606 – Math - Statistics and Probability

Problem.

A player tosses 3 fair coins. He wins Rs.500 if 3 heads appear, Rs.300 if 2 heads appear, Rs.100 if 1 head occurs. On the other hand, he loses Rs.1500 if 3 tails occur. Find the expected gain of the player and variance.

Solution.

The probability that head appears equals $\frac{1}{2} = 0.5$. The probability that tail appears equals $\frac{1}{2} = 0.5$.

The probability that 3 heads appear equals $0.5^3 = 0.125$ by Bernoulli trial.

The probability that 2 heads appear equals $\binom{3}{2} 0.5^2 0.5 = 0.375$ by Bernoulli trial.

The probability that 1 heads appear equals $\binom{3}{1} 0.5^2 0.5 = 0.375$ by Bernoulli trial.

The probability that 0 heads appear equals $\binom{3}{0} 0.5^2 0.5 = 0.125$ by Bernoulli trial.

Head occurs	3	2	1	0
Gain	500	300	100	-1500
Probability	0.125	0.375	0.375	0.125

$$\text{Expected gain } E = 0.125 \cdot 500 + 0.375 \cdot 300 + 0.375 \cdot 100 + 0.125 \cdot (-1500) = 25.$$

$$\text{Variance} = 0.125 \cdot (500 - E)^2 + 0.375 \cdot (300 - E)^2 + 0.375 \cdot (100 - E)^2 + 0.125 \cdot (-1500 - E)^2 = 346953.125.$$