

Answer on Question #45605 – Math – Calculus

The function $f(x) = [x]^n - [x^n]$, for $n > \text{or} = 2$ (where $[a]$ denotes the greatest integer less than or equal to 'a'), is discontinuous at all points of:

1. all integers
2. all integers except 0 and 1
3. all integers except 0
4. all integers except 1

answer: (4)

{ Please give me a descriptive answer rather than just solution and thanks in advance }

Solution.

$f(x) = 0$ for $x \in I$ (all integers).

If $0 < x < 1$ then $0 < x^n < 1$ so,

$[x] = 0$ and $[x^n] = 0 \rightarrow f(x) = 0$ for $0 < x < 1$.

If $1 < x < 2^{\frac{1}{n}}$ then $1 < x^n < 2 \rightarrow [x] = 1$ and $[x^n] < 1$

thus $f(x) = 0$ if $1 < x < 2^{\frac{1}{n}}$.

So, $f(x) = 0$ when $0 \leq x < 2^{\frac{1}{n}}$.

Therefore, $f(x)$ is continuous at $x = 1$.

To the left of any integer value $m \neq 1$ but close to m , $f(x) \neq 0$ but right to the m but close to , $f(x) = 0$. Hence, $f(x)$ is discontinuous for all $m \in I$ except $[1]$

(case 4.).