

Answer on Question #45602 – Math – Statistics and Probability

Question.

In a test on electric bulbs, it was found that the life time of a particular brand was normally distributed with an average life of 2000 hours and S.D. of 60 hours. If a firm purchases 2500 bulbs, find the number of bulbs that are likely to last for (i) more than 2100 hours, (ii) less than 1950 hours and (iii) between 1900 and 2100 hours.

Solution.

Let ξ be the life time of a particular brand. Then $\eta = \frac{\xi - 2000}{60}$ has a standard normal distribution i. e. $P(\eta < x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{t^2}{2}} dt$. $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{t^2}{2}} dt$ is a tabulated function of Laplace.

$$\begin{aligned} \text{(i)} \quad P(\xi > 2100) &= 1 - P(\xi < 2100) = 1 - P\left(\frac{\xi - 2000}{60} < 1.67\right) = 1 - (0.5 + \Phi(1.67)) = \\ &= 1 - (0.5 + 0.45254) = 0.04746. \text{ The corresponding number of bulbs is } 2500 \cdot 0.04746 \approx 119 \text{ bulbs.} \end{aligned}$$

$$\text{(ii)} \quad P(\xi < 1950) = P\left(\frac{\xi - 2000}{60} < -0.83\right) = 0.5 - \Phi(0.83) = 0.5 - 0.29673 = 0.20327. \text{ The corresponding number of bulbs is } 2500 \cdot 0.20327 \approx 508 \text{ bulbs.}$$

$$\text{(iii)} \quad P(1900 < \xi < 2100) = P\left(-1.67 < \frac{\xi - 2000}{60} < 1.67\right) = 2 \cdot \Phi(1.67) = 2 \cdot 0.45254 = 0.90508. \text{ The corresponding number of bulbs is } 2500 \cdot 0.90508 \approx 2263 \text{ bulbs.}$$

Answer.

(i) 119 bulbs

(ii) 508 bulbs

(iii) 2263 bulbs