Answer on Question #45588 - Math – Calculus

What is the easiest asymptote to find (horizontal, vertical, oblique)?

Solution

1. Horizontal asymptote is nothing else but particular case of an oblique asymptote.

$y = ax + b -$ *oblique* asymptote;

$y = b$ (here $a = 0$) $-$ *horizontal* asymptote, where $a, b -$ finite constants.

The ways of determination for both types of the asymptotes mentioned above are similar.

For *oblique* asymptote: $\lim_{x\to-\infty}[g(x) - (ax + b)] = 0$, $\lim_{x\to\infty}[g(x) - (ax + b)] = 0$.

For *horizontal* asymptote: $\lim_{x\to-\infty}[g(x) - b] = 0$, $\lim_{x\to\infty}[g(x) - b] = 0$.

Note that the *horizontal* asymptote of the graph of the function $y = g(x)$ requires only straightforward calculation of the limit: $b = \lim_{x\to-\infty} g(x)$ or $b = \lim_{x\to\infty} g(x)$. The *oblique* asymptote of the graph of the function $y = g(x)$ requires additionally some transformations of the function $g(x)$ to define parameter a or calculations of the following limits

$$a = \lim_{x\to+\infty}\frac{g(x)}{x}, b = \lim_{x\to+\infty}(g(x) - ax) \text{ or } a = \lim_{x\to-\infty}\frac{g(x)}{x}, b = \lim_{x\to-\infty}(g(x) - ax).$$

Hence, the horizontal asymptote is easier to find than the oblique asymptote.

2. Let's compare the horizontal and vertical asymptotes.

$x = c -$ *vertical* asymptote, where $c -$ finite constant.

Conditions of *vertical* asymptote: $\lim_{x\to c^-} g(x) = \pm\infty$ or $\lim_{x\to c^+} g(x) = \pm\infty$.

"To find *vertical* asymptote" means "to find such finite constant c, which satisfies at least one of the conditions above". Instead of straightforward calculation of the limit, this task can be extremely hard or even impossible. For example: $g(x) = \frac{1}{x^5+x^4+x^3+x^2+x-1}$, polynomial at denominator has at least one real solution c, which is at the same time the c for asymptote's equation we looking for. But as soon as it is 5th-order polynomial, we unable to calculate c analytically (just approximately by numerical methods).

Thus, *horizontal* asymptote is the easiest asymptote to find.