Answer on Question #45584 – Math – Complex Analysis

Problem.

If arg((z-1)/(z+1)) =45(pi/4) , show that the locus z in the complex plain is a circle.

Solution.

If $w = \frac{z-1}{z+1}$, then $w = |w|(\cos(\arg(w)) + \sin(\arg(w))) = |w|\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = r + ir$ where $r \in \mathbb{R}^+$. Then $\frac{z-1}{z+1} = \frac{(x-1)+iy}{(x+1)+iy} = \frac{((x-1)+iy)((x+1)-iy)}{((x+1)+iy)((x+1)-iy)} = \frac{x^2+y^2-1+2iy}{(x+1)^2+y^2} = r + ir$. Hence $\arg\left(\frac{z-1}{z+1}\right) = 45°$ if and only if $x^2 + y^2 - 1 = 2y$ or $x^2 + (y-1)^2 = 2$. $x^2 + (y-1)^2 = 2$ is the equation of circle with center $(0,1)$ and radius $\sqrt{2}$.