Answer on Question #45579 – Math – Statistics and Probability

Problem.

The administration department of PIMS Hospital Islamabad surveyed the number of days 400 randomly chosen patients stayed in the hospital after an operation.
The data are:
Days (of hospital say): 1-3 4-6 7-9 10-12 13-15 16-18 19-21 22-24
Frequency: 36 18 88 42 18 18 8 10
a) Calculate the standard deviation and mean.
b) According to Chebyshev's theorem, how many stays should be between 0 and 17 days? How many are actually in that interval?
c) How many stays can we expect between 0 and 17 days?

Remark:

I suppose that there is mistake in a problem, as $36 + 18 + 88 + 42 + 18 + 18 + 8 + 10 = 238 \neq 400$. I suppose that there the sample has size 238.

Solution:

a) From the data

Days	1-3	4-6	7-9	10-12	13-15	16-18	19-21	22-24	Total
Midpoint	2	5	8	11	14	17	20	23	
Frequency	36	18	88	42	18	18	8	10	238
Midpoint^2	4	25	64	121	196	289	400	529	

$$\text{Mean} = \frac{2 \cdot 36 + 5 \cdot 18 + 8 \cdot 88 + 11 \cdot 42 + 14 \cdot 18 + 17 \cdot 18 + 20 \cdot 8 + 23 \cdot 10}{238} \approx 9.56.$$

$$\text{Mean of sum of midpoints squares} = \frac{4 \cdot 36 + 25 \cdot 18 + 64 \cdot 88 + 121 \cdot 42 + 196 \cdot 18 + 289 \cdot 18 + 400 \cdot 8 + 529 \cdot 10}{238} \approx 119.86.$$

$$\text{Standard deviation} = \text{SD} = \sqrt{\text{Mean of sum of midpoints squares} - \text{Mean}^2} = 1.38.$$

b) Chebyshev's theorem or Chebyshev's inequality says that $1 - \frac{1}{k^2}$ of the distribution's values are within k standard deviations of the mean.
0-17 range equals $\text{Mean} \pm 7\text{SD}$ ($9.56 \pm 7 \cdot 1.38$), so we can expect $1 - \frac{1}{7^2} = 0.98 = 98\%$ of 238 or at least 233 stays.
There are actually from $36 + 18 + 88 + 42 + 18 = 202$ to $202 + 18 = 220$ stays between 0 and 17 days.
c) By the empirical rule in the range $\text{Mean} \pm 7\text{SD}$ will lie 99.99% of stays (237 stays).