# Answer on Question #45577 – Math – Statistics and Probability

**Problem.**

General Co. has designed a new tire, and they don't know what the average amount of tread life is going to be. The tread life is normally distributed with a standard deviation of 295.5 kilometers.
**i)** If the company samples 800 tires and records their tread life, what is the probability the sample mean is between the true mean and 350 kilometers over the true mean?
**ii)** How large a sample must be taken to be 95 percent sure the sample mean will be within 150 kilometers of the true mean?

**Solution.**

Suppose that tread life is normally distributed with mean $\mu$ and standard deaviation $\sigma = 295.5$ ($N(\mu, \sigma)$).
**i)** The sample of $n = 800$ tires (random variable $X$) has distribution $N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$.
The probability that the sample mean is between the true mean and 350 kilometers over the true mean is

$$P(\mu < X < \mu + 350).$$

The corresponding transformation formula is $Z = \frac{X-\mu}{\sigma/\sqrt{n}}$, where $Z$ is distributed normally, $Z \sim N(0,1)$.

$$P(\mu < X < \mu + 350) = P\left(0 < Z < \frac{350}{295.5 \cdot \sqrt{800}}\right) = P(0 < Z < 0.041) \approx 0.0164.$$

**ii)** $z^*$ for 95% confidence level equals $z^* = 1.96$. Then the confidence interval is

$$\left(\mu - z^* \frac{\sigma}{\sqrt{n}}, \mu + z^* \frac{\sigma}{\sqrt{n}}\right)$$

Then the sample mean will be within 150 kilometers of the true mean if $z^* \frac{\sigma}{\sqrt{n}} < 150$ or $n > \left(\frac{1.96 \cdot 295.5}{150}\right)^2 \approx 14.9$. Hence $n \geq 15$.
**Answer:** i) $0.0164$. ii) $n \geq 15$.