

## Answer on Question #45574 – Math – Algebra

**Question.** Prove it by the principal of Mathematical Induction:

$$\sin x + \sin 3x + \cdots + \sin(2n+1)x = \frac{\sin^2(n+1)x}{\sin x}.$$

**Proof.**

*Basis of induction.* Let  $n = 1$ .

Then we have in the LHS

$$\sin x + \sin 3x = 2\sin 2x \cdot \cos x = 4\sin x \cdot \cos^2 x.$$

In the RHS we have

$$\frac{\sin^2 2x}{\sin x} = \frac{4\sin^2 x \cdot \cos^2 x}{\sin x} = 4\sin x \cdot \cos^2 x.$$

We have the true identity. The basis of induction is proved.

*Step of induction.* Assume that the formula

$$\sin x + \sin 3x + \cdots + \sin(2n+1)x = \frac{\sin^2(n+1)x}{\sin x}$$

holds for some  $n = k$ . We prove that the same formula holds for  $n = k + 1$ .

$$\begin{aligned} & [\sin x + \sin 3x + \cdots + \sin(2k+1)x] + \sin(2k+3)x = \frac{\sin^2(k+1)x}{\sin x} + \sin(2k+3)x = \\ &= \frac{\sin^2(k+1)x + \sin(2k+3)x \cdot \sin x}{\sin x} = \frac{1}{\sin x} \left[ \frac{1 - \cos(2k+2)x}{2} + \frac{1}{2} (\cos(2k+2)x - \cos(2k+4)x) \right] = \\ &= \frac{1}{\sin x} \cdot \frac{1}{2} (1 - \cos(2k+2)x + \cos(2k+2)x - \cos(2k+4)x) = \frac{1}{\sin x} \cdot \frac{1 - \cos(2k+4)x}{2} = \\ &= \frac{\sin^2(k+2)x}{\sin x}. \end{aligned}$$

Finally we have  $\sin x + \sin 3x + \cdots + \sin(2k+1)x + \sin(2k+3)x = \frac{\sin^2(k+2)x}{\sin x}$ . The step of induction is proved and the formula  $\sin x + \sin 3x + \cdots + \sin(2n+1)x = \frac{\sin^2(n+1)x}{\sin x}$  is proved.