Answer on Question # 45525 – Math – Statistics and Probability

Three cooks, A, B and C bake a special kind of cake, and with respective probabilities 0.02, 0.03, and 0.05 it fails to rise. In the restaurant where they work, A bake 50 percent of these cakes, B 30 percent and C 20 percent. What proportion of failures is caused by A.

Solution:

We start to solve with definition of the probability events applying to our problem.

The probability of an event A occurring when it is known that some event A has occurred is called a conditional probability and is denoted by $\mathrm{P}(\mathrm{A}|\mathrm{B})$. The symbol $\mathrm{P}(\mathrm{A}|\mathrm{B})$ is usually read the probability that A occurs given that B occurs or simply the probability of A given B.

The conditional probability of A, given B, denoted by $\mathrm{P}(\mathrm{A}|\mathrm{B})$, is defined the following formula.

$$\mathrm{P}(\mathrm{A}|\mathrm{B}) = \frac{\mathrm{P}(\mathrm{A}\cap\mathrm{B})}{\mathrm{P}(\mathrm{B})} \text{ if } \mathrm{P}(\mathrm{B}) > 0$$

Note the given values accordingly to the condition of the task. The cook A bake a 50% of these cakes with probability P(A)=0.5, the cook B bake a 30% of cakes with probability P(B)=0.3 and the cook C bake a 20% of cakes with probability P(C)=0.2.

Let F be the event that the cake fails to rise. Then we can write the probability with takes into account this condition.

$$\mathrm{P}(\mathrm{F}|\mathrm{A}) = 0.02, \mathrm{P}(\mathrm{F}|\mathrm{B}) = 0.03 \text{ and } \mathrm{P}(\mathrm{F}|\mathrm{C}) = 0.05$$

To solve our problem we apply the Bayes' Theorem. Accordingly to the theorem it should be noted.

Let the events A_1, A_2,..., A_k forms a partition of the space S such that $\mathrm{P}(\mathrm{A}_\mathrm{j}) > 0$, for $\mathrm{j} = 1, .., \mathrm{k}$, and let B be any event such that $\mathrm{P}(\mathrm{B}) > 0$. Then, for $\mathrm{j} = 1, \ldots, \mathrm{k}$,

$$\mathrm{P}(\mathrm{A}_\mathrm{j}|\mathrm{B}) = \frac{\mathrm{P}(\mathrm{A}_\mathrm{j})\mathrm{P}(\mathrm{B}|\mathrm{A}_\mathrm{j})}{\mathrm{P}(\mathrm{B})} = \frac{\mathrm{P}(\mathrm{A}_\mathrm{j})\mathrm{P}(\mathrm{B}|\mathrm{A}_\mathrm{j})}{\sum_{i=1}^{k}\mathrm{P}(\mathrm{A}_\mathrm{i})\mathrm{P}(\mathrm{B}|\mathrm{A}_\mathrm{i})}$$

Apply the formula noted above to solve our problem.

$$\mathrm{P}(\mathrm{A}|\mathrm{F}) = \frac{\mathrm{P}(\mathrm{A}\cap\mathrm{F})}{\mathrm{P}(\mathrm{F})}$$

Where $\mathrm{P}(\mathrm{F})$ is equal to the following formula.

$$\mathrm{P}(\mathrm{F}) = \mathrm{P}(\mathrm{A}\cap\mathrm{F}) + \mathrm{P}(\mathrm{B}\cap\mathrm{F}) + \mathrm{P}(\mathrm{C}\cap\mathrm{F}) = \mathrm{P}(\mathrm{A})\mathrm{P}(\mathrm{F}|\mathrm{A}) + \mathrm{P}(\mathrm{B})\mathrm{P}(\mathrm{F}|\mathrm{B}) + \mathrm{P}(\mathrm{C})\mathrm{P}(\mathrm{F}|\mathrm{C})$$

According to the condition of the task we have all data, so we can substitute into the formula noted above.

$$\mathrm{P(F)} = (0.5 \cdot 0.02) + (0.3 \cdot 0.03) + (0.2 \cdot 0.05) = 0.01 + 0.009 + 0.01 = 0.029$$

Now we can substitute the obtained value into the formula. We know that

$$\mathrm{P(A \cap F)} = \mathrm{P(A)P(F|A)} = 0.01$$

So, we can find the value of $\mathrm{P(A|F)}$.

$$\mathrm{P(A|F)} = \frac{\mathrm{P(A \cap F)}}{\mathrm{P(F)}} = \frac{0.01}{0.029} \approx 0.344827$$

Finally we can write that $\mathrm{P(A|F)} = 0.34482$

Answer: The proportion of failures is caused by A is equal to $\mathbf{P(A|F) = 0.34482}$ (approximately 34%).