Answer on Question #45522 - Math – Statistics and Probability

Normal distribution:

$$f(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Where $\mu -$ mean of the distribution, $\sigma -$ standard deviation, $f(x,\mu,\sigma) -$ probability density function, $x -$ variable.

Input of the task: $\sigma = 0.02$ inch, $P(3.5 < x) \leq 0.09$ (9% corresponds to $\frac{9}{100} = 0.09$), $\mu - ?$

$P(3.5 < x) -$ probability of x to be greater than 3.5 inches; $x -$ diameter of the “can tops”.

$$P(3.5 < x) = \int_{3.5}^{\infty} f(x,\mu,\sigma)\,dx = \int_{3.5}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx \leq 0.09$$

$\sigma = 0.02\ (inch)\ ;\ \sigma^2 = 0.02^2 = 0.0004\ (inch^2);$

$$\int_{3.5}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx = \int_{3.5}^{\infty} \frac{1}{0.02\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\times 0.0004}}\,dx = \frac{1}{0.02\sqrt{2\pi}} \int_{3.5}^{\infty} e^{-\frac{(x-\mu)^2}{0.0008}}\,dx \leq 0.09$$

$$\int_{3.5}^{\infty} e^{-\frac{(x-\mu)^2}{0.0008}}\,dx \leq 0.09 \times 0.02\sqrt{2\pi} \approx 0.004512 = 4.512 \times 10^{-3}$$

Consider $t = \frac{x-\mu}{\sqrt{0.0008}},\ dt = \frac{dx}{\sqrt{0.0008}},\ dx = \sqrt{0.0008} \times dt \approx 0.0283dt.$

Upper boundary: $\lim\limits_{x\to\infty} \frac{x-\mu}{\sqrt{0.0008}} = \infty;$ lower boundary: $\frac{x-\mu}{\sqrt{0.0008}}\Big|_{3.5} \approx \frac{3.5-\mu}{0.0283} \approx 35.36(3.5-\mu);$

Thus, $\int_{3.5}^{\infty} e^{-\frac{(x-\mu)^2}{0.0008}}\,dx = 0.0283 \int_{35.36(3.5-\mu)}^{\infty} e^{-t^2}\,dt \leq 4.512 \times 10^{-3};$

Consider $a = 35.36(3.5-\mu).$

$$0.0283 \int_{a}^{\infty} e^{-t^2}\,dt \leq 4.512 \times 10^{-3}$$

$$\int_{a}^{\infty} e^{-t^2}\,dt \leq 159.435 \times 10^{-3} \approx 0.1594$$

There is no elementary indefinite integral for $\int e^{-t^2}\,dt$, hence the only way is to use methods of numerical integration. Result: $a \geq 0.94836.$

Thus, $a = 35.36(3.5-\mu) \geq 0.94836;\ -\mu \geq \frac{0.94836}{35.36} - 3.5;\ \mu \leq 3.5 - \frac{0.94836}{35.36};$

$\mu \leq 3.5 - 0.02682;\quad \mu \leq 3.47318;$