Answer on Question #45517 – Math - Statistics and Probability

Problem.

(a) A player tosses 3 fair coins. He wins Rs.500 if 3 heads appear, Rs.300 if 2 heads appear, Rs.100 if 1 head occurs. On the other hand, he loses Rs.1500 if 3 tails occur. Find the expected gain of the player and variance.

(b) The trouble shooting capability of an IC chip in a circuit is a random variable X whose distribution function is given by where x denote the number of years. Find the probability that the IC chip will work properly (i) less than 8 years (ii) beyond 8 years (iii) between 5 to 7 years

Solution.

(a) The probability that head appears equals $\frac{1}{2} = 0.5$. The probability that tail appears equals $\frac{1}{2} = 0.5$.

The probability that 3 heads appear equals $0.5^3 = 0.125$ by Bernoulli trial.

The probability that 2 heads appear equals $\binom{3}{2} 0.5^2 0.5 = 0.375$ by Bernoulli trial.

The probability that 1 heads appear equals $\binom{3}{1} 0.5^2 0.5 = 0.375$ by Bernoulli trial.

The probability that 0 heads appear equals $\binom{3}{0} 0.5^2 0.5 = 0.125$ by Bernoulli trial.

Head occurs	3	2	1	0
Gain	500	300	100	−1500
Probability	0.125	0.375	0.375	0.125

$$\text{Expected gain } E = 0.125 \cdot 500 + 0.375 \cdot 300 + 0.375 \cdot 100 + 0.125 \cdot (-1500) = 25 = E.$$

$$\text{Variance} = 0.125 \cdot (500 - E)^2 + 0.375 \cdot (300 - E)^2 + 0.375 \cdot (100 - E)^2 + 0.125 \cdot (-1500 - E)^2 = 346953.125.$$

(b) The part of question is missed "whose distribution function is given by f(x)=…, where x denote the number of years". Here f(x) is the probability distribution function, F(x) is the cumulative distribution function.

(i) The probability that that the IC chip will work properly less than 8 years is $F(8)$.

(ii) The probability that that the IC chip will work properly beyond 8 years is $1 - F(8)$.

(iii) The probability that that the IC chip will work properly between 5 to 7 years 8 is $F(7) - F(5)$.