Answer on Question #45511 – Math – Statistics and Probability

Problem.

The sales in a two wheeler showroom is exponentially distributed with mean equal to 4. If two days are selected at random, what is the probability that
a) on both days the sales is over 5 units
b) the sale is over 5 units on at least one of the two days.

Solution.

The probability density function of an exponential distribution is

$$f(x; \lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0; \\ 0 & x < 0, \end{cases}$$

where λ is parameter.

The mean of an exponentially distributed random variable with rate parameter λ is equal to $\frac{1}{\lambda}$.

Hence if random variable is exponentially distributed with mean equal to 4 $\left(\frac{1}{\lambda} = 4\right)$, then this random variable has density function

$$f\left(x; \frac{1}{4}\right) = \begin{cases} \dfrac{e^{-\frac{x}{4}}}{4} & x \geq 0; \\ 0 & x < 0, \end{cases}$$

The probability that the sales on one day is over 5 units is equal to

$$\int_5^{+\infty} f\left(x; \frac{1}{4}\right) dx = \int_5^{+\infty} \frac{e^{-\frac{x}{4}}}{4} \, dx = -\int_5^{+\infty} e^{-\frac{x}{4}} \, d\left(-\frac{x}{4}\right) = -e^{-\frac{x}{4}} \Big|_5^{+\infty} \approx 0.287.$$

The probability that the sale on one day is below 5 units equals
$$1 - 0.287 = 0.713,$$
as probability of complementary event.

The probability that the sales on both days is over 5 unit is equal to
$$0.287^2 \approx 0.082,$$
as probability of intersection of two events.

The probability that the sale is over 5 units on at least one of the two days is equal to
$$1 - 0.713^2 \approx 0.491.$$
as probability of complementary event.

Answer: a) 0.082, b) 0.491.