Answer on Question #45493 – Math - Integral Calculus

y suffix n = intregation sinnx/sinx dx

then show that

y= 2sin (n-1)x/(n-1) +y suffix n-2

HENCE evaluate

intregation limit 0 to pie /2

sin7x/sinx

Solution.

$$y = suffix\,(n) = \int \frac{sinnx}{sinx}dx = \int \frac{sin[(n-2)x + 2x]}{sinx}dx =$$

$$= \int \frac{sin(n-2)x * (1 - 2sin^2x) + 2cos(n-2)x * sinx * cosx}{sinx}dx =$$

$$= suffix\,(n-2) + \int -2sin(n-2)x * sinx + 2cos(n-2)x * cosxdx =$$

$$= suffix\,(n-2) - 2\int cos(n-1)xdx = \frac{2sin(n-1)x}{n-1} + suffix\,(n-2).$$

So,

$$suffix(7) = suffix(5) + \frac{2sin6x}{6} = suffix(3) + \frac{2sin4x}{4} + \frac{sin6x}{3} =$$

$$= suffix(1) + \frac{2sin2x}{2} + \frac{2sin4x}{4} + \frac{sin6x}{3} = \int dx + sin2x + \frac{sin4x}{2} + \frac{sin6x}{3} =$$

$$= x + \; sin2x + \frac{sin4x}{2} + \frac{sin6x}{3} + const.$$

Thus: $\int_0^{\frac{\pi}{2}} \frac{sin7x}{sinx}dx = \left(x + \; sin2x + \frac{sin4x}{2} + \frac{sin6x}{3}\right)\Big|_{x=0}^{x=\frac{\pi}{2}} = \frac{\pi}{2}.$