Answer on Question #45405 – Math - Abstract Algebra

Problem.

Let D(subscript12) =({x,y : x^2 = e ; y^6 = e ; xy =(y^-1) x}) a) Which of the following subsets are subgroups of D(subscript12) ? Justify your answer. i) {x,y,xy,y^2,y^3,e} ii) {xy,xy^2,y^2,e} iii) {x,y^3,xy^3,e}

Solution.

G is supgroup of D_{12} if and only if it is closed under product and inverses.

i) Suppose $G_1 = \{x, y, xy, y^2, y^3, e\}$ is a subgroup of D_{12}. Then $y^2 \cdot y^3 = y^5 \in G_1$. Hence y^5 should be equal to either x or y or xy or y^2 or y^3.

If $x = y^5$, then $e = x^2 = y^{10} = y^4$ or $y^2 = e$, what is impossible (the order of y is 6).

If $y = y^5$, then $e = y^4$ or $y^2 = e$, what is impossible (the order of y is 6).

If $xy = y^5$, then $x = y^4$ or $e = x^2 = y^8 = y^3$, what is impossible (the order of y is 6).

If $y^2 = y^5$, then $e = y^3$, what is impossible (the order of y is 6).

If $y^3 = y^5$, then $e = y^2$, what is impossible (the order of y is 6).

Hence y^5 isn't equal to neither x nor y nor xy nor y^2 nor y^3 nor e. Therefore G_1 isn't a subgroup.

ii) Suppose $G_2 = \{xy, xy^2, y^2, e\}$ is a subgroup of D_{12}. Then $y^2 \cdot y^2 = y^4 \in G_2$. Hence y^4 should be equal to either xy or xy^2 or y^2.

If $xy = y^4$, then $x = y^3$. Then $G_2 = \{y^4, y^5, y^2, e\}$, what is impossible ($y^5$ doesn't have inverse).

If $xy^2 = y^4$, then $x = y^2$, $e = x^2 = y^4$ or $y^2 = e$, what is impossible (the order of y is 6).

If $y^2 = y^5$, then $e = y^3$, what is impossible (the order of y is 6).

Hence y^4 isn't equal to neither xy nor xy^2 nor y^2. Therefore G_2 isn't a subgroup.

iii) $G_3 = \{x, y^3, xy^3, e\}$.

$x^{-1} = x$, as $x^2 = e$;

$(y^3)^{-1} = y^3$, as $y^6 = e$;

$(xy^3)^{-1} = xy^3$, as $(xy^3)^2 = xy^3 \cdot y^3x = xy^6x = x^2 = e$ ($xy^3 = (y^3)^{-1}\ x = y^3x$);

$e^{-1} = e$;

Hence G_3 is closed under inverses.

$y^3 \cdot x = x \cdot y^3 \in G_3$;

$x \cdot xy^3 = x^2y^3 = y^3 \in G_3$;

$xy^3 \cdot x = y^3x \cdot x = y^3 \in G_3$ ($xy^3 = (y^3)^{-1}\ x = y^3x$);

$y^3 \cdot xy^3 = y^3 \cdot y^3x = y^6x = x \in G_3$;

$xy^3 \cdot y^3 = xy^6 = x \in G_3$;

Hence G_3 is closed under products.

Therefore G_3 is a subgroup.

Answer: iii).