Answer on Question #45400 – Math - Calculus

integrate 1/sqrt(sin^4(x) +1) dx

Solution

$$I = \int \frac{dx}{\sqrt{sin^4x + 1}}$$

This function cannot be integrated. It's a nonelementary integral.

To evaluate this integral we can use Maclauren series.

$$f(x) = \frac{1}{\sqrt{sin^4x + 1}}\ , \qquad f(0) = 1\ ,$$

$$f'(x) = -\frac{2sin^3xcosx}{(sin^4x + 1)^{\frac{3}{2}}} = -2A(x)sin^3x, \qquad where\ A(x) = \frac{cosx}{(sin^4x + 1)^{\frac{3}{2}}},$$

$$f'(0) = 0,$$

$$f''(x) = -2A'(x)sin^3x - 6A(x)sin^2xcosx,$$

$$f''(0) = 0,$$

$$f'''(x) = -2sin^2x(A''(x)sinx + 6A'(x)cosx + 3A(x)sinx) -$$

$$-12A(x)sinxcos^2x = -2B(x)sin^2x - 12A(x)sinxcos^2x,$$

$$f'''(0) = 0,$$

$$f^{IV}(x) = -2sinx(B'(x)sinx + 2B(x)cosx + 6A'(x)cos^2x -$$

$$-12sinxcosx) - 12A(x)cos^3x,$$

$$f^{IV}(0) = -12A(0) = -12$$

So, $f(x) = f(0) + f'(0)x + \frac{1}{2}f''(0)x^2 + \frac{1}{6}f'''(0)x^3 + \frac{1}{24}f^{IV}(0)x^4 + O(x^5)$

Thus $f(x) = \frac{1}{\sqrt{sin^4 x+1}} = 1 - \frac{x^4}{2} + O(x^5).$

Finally,

$$I = \int \frac{dx}{\sqrt{sin^4 x + 1}} = \int (1 - \frac{x^4}{2} + O(x^6))\, dx = x - \frac{x^5}{10} + O(x^6).$$