

Answer on Question #45382 – Math – Analytical Geometry

Find the center, vertices, and foci of the ellipse with equation $3x^2 + 6y^2 = 18$

Solution:

To be able to read any information from this equation, we'll need to rearrange it to get "=1", so I'll divide through by 18.

This gives

$$\begin{aligned} 3x^2 + 6y^2 &= 18 \\ \frac{3x^2}{18} + \frac{6y^2}{18} &= \frac{18}{18} \\ \frac{x^2}{6} + \frac{y^2}{3} &= 1 \end{aligned}$$

Since $x^2 = (x - 0)^2$ and $y^2 = (y - 0)^2$, the equation above is really:

$$\frac{(x - 0)^2}{6} + \frac{(y - 0)^2}{3} = 1$$

Then the center is at $(h, k) = (0, 0)$. I know that the a^2 is always the larger denominator (and b^2 is the smaller denominator), and this larger denominator is under the variable that parallels the longer direction of the ellipse. Since 6 is larger than 3, then $a^2 = 6$, $a = \pm\sqrt{6}$, and this ellipse is wider (paralleling the x-axis) than it is tall. The value of a also tells me that the vertices are $\sqrt{6}$ units to either side of the center, at $(-\sqrt{6}, 0)$ and $(\sqrt{6}, 0)$.

Let's find co-vertices of the ellipse:

$$\begin{aligned} b^2 &= 3 \\ b &= \pm\sqrt{3} \end{aligned}$$

Co-vertices: $(-\sqrt{3}, 0)$ and $(\sqrt{3}, 0)$.

To find the foci, we need to find the value of c . From the equation, I already have a^2 and b^2 , so:

$$\begin{aligned} a^2 - c^2 &= b^2 \\ 6 - c^2 &= 3 \\ c^2 &= 3 \\ c &= \sqrt{3} \end{aligned}$$

Then the value of c is 3, and the foci are three units to either side of the center, at $(-\sqrt{3}, 0)$ and $(\sqrt{3}, 0)$

Answer: center $(0, 0)$,

vertices: $(-\sqrt{6}, 0)$ and $(\sqrt{6}, 0)$, $(-\sqrt{3}, 0)$ and $(\sqrt{3}, 0)$.

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