

Answer on Question #45330 – Math – Statistics and Probability

Question. The joint probability function of two discrete variables X and Y is given by $f(x, y) = c(2x + y)$, where x and y can assume all integral values such that $0 \leq x \leq 2$, $0 \leq y \leq 3$, and $f(x, y) = 0$ otherwise. Find

(i) The value of the constant c

(ii) $P(X = 2, Y = 1)$

(iii) $P(X \geq 1, Y \leq 2)$

(iv) Marginal probability functions of X and Y . Check whether X and Y are independent.

Solution.

(i) Use the next property of $f(x, y)$: $\sum_{i=0}^2 \sum_{j=0}^3 f(i, j) = 1$. We have:

$$\begin{aligned} \sum_{i=0}^2 (f(i, 0) + f(i, 1) + f(i, 2) + f(i, 3)) &= f(0, 0) + f(0, 1) + f(0, 2) + f(0, 3) + \\ &+ f(1, 0) + f(1, 1) + f(1, 2) + f(1, 3) + f(2, 0) + f(2, 1) + f(2, 2) + f(2, 3) = \\ &= 0 \cdot c + c + 2c + 3c + 2c + 3c + 4c + 5c + 4c + 5c + 6c + 7c = 42c = 1 \Rightarrow c = \frac{1}{42}. \end{aligned}$$

$$(ii) f(x, y) = \frac{1}{42}(2x + y). P(X = 2, Y = 1) = f(2, 1) = \frac{1}{42} \cdot 5 = \frac{5}{42}.$$

$$\begin{aligned} (iii) P(X \geq 1, Y \leq 2) &= \sum_{i=1}^2 \sum_{j=0}^2 f(i, j) = \sum_{i=1}^2 (f(i, 0) + f(i, 1) + f(i, 2)) = f(1, 0) + \\ &+ f(1, 1) + f(1, 2) + f(2, 0) + f(2, 1) + f(2, 2) = \frac{1}{42}(2 + 3 + 4 + 4 + 5 + 6) = \frac{4}{7}. \end{aligned}$$

$$\begin{aligned} (iv) f_X(x) &= \sum_{j=0}^3 f(x, j) = f(x, 0) + f(x, 1) + f(x, 2) + f(x, 3) = \\ &= \frac{1}{42}(2x + 2x + 1 + 2x + 2 + 2x + 3) = \frac{1}{42}(8x + 6) = \frac{1}{21}(4x + 3), 0 \leq x \leq 2. \end{aligned}$$

$$\begin{aligned} f_Y(y) &= \sum_{i=0}^2 f(i, y) = f(0, y) + f(1, y) + f(2, y) = \frac{1}{42}(y + 2 + y + 4 + y) = \\ &= \frac{1}{42}(3y + 6) = \frac{1}{14}(y + 2), 0 \leq y \leq 3. \end{aligned}$$

Since $f_X(x)f_Y(y) = \left(\frac{4}{21}x + \frac{1}{7}\right)\left(\frac{1}{14}y + \frac{1}{7}\right) = \frac{2}{147}xy + \frac{4}{147}x + \frac{1}{98}y + \frac{1}{49} \neq \frac{1}{21}x + \frac{1}{42}y = f(x, y)$, X and Y are not independent.

Answer.

$$(i) c = \frac{1}{42}$$

$$(ii) P(X = 2, Y = 1) = \frac{5}{42}$$

$$(iii) P(X \geq 1, Y \leq 2) = \frac{4}{7}$$

(iv) $f_X(x) = \frac{1}{21}(4x + 3), 0 \leq x \leq 2$

$$f_Y(y) = \frac{1}{14}(y + 2), 0 \leq y \leq 3.$$

X and Y are not independent.