Answer on Question #45290 - Math - Calculus

$$4 \times \left(\frac{\sin\left(\frac{x}{4}\right) - \cos\left(\frac{x}{4}\right)}{\sin\left(\frac{x}{4}\right) + \cos\left(\frac{x}{4}\right)} \times \sqrt{1 + \sin\left(\frac{x}{2}\right)} \right) + C,$$

where C – constant.

Solution

$$\int \left(1 + \sin\left(\frac{x}{2}\right)\right)^{\frac{1}{2}} dx$$

$$g = \frac{x}{2}; \;\; dx = 2dg;$$

$$\int \left(1 + \sin\left(\frac{x}{2}\right)\right)^{\frac{1}{2}} dx = 2 \int (1 + \sin(g))^{\frac{1}{2}} dg$$

Tangent half-angle substitution:

$$t = \tan\left(\frac{g}{2}\right); \quad \sin(g) = \frac{2t}{1+t^2}; \quad dg = \frac{2dt}{1+t^2};$$

$$2 \int (1 + \sin(g))^{\frac{1}{2}} dg = 4 \int \frac{\left(1 + \frac{2\mathrm{t}}{1+\mathrm{t}^2}\right)^{\frac{1}{2}}}{1+t^2} dt = 4 \int \frac{\left(\frac{1 + 2\mathrm{t} + \mathrm{t}^2}{1+\mathrm{t}^2}\right)^{\frac{1}{2}}}{1+t^2} dt =$$

$$4 \int \frac{\left(\frac{(1+t)^2}{1+t^2}\right)^{\frac{1}{2}}}{1+t^2} dt = 4 \int \frac{\sqrt{(1+t)^2}}{(1+t^2)^{\frac{3}{2}}} dt = 4 \int \frac{(1+t)sgn(1+t)}{(1+t^2)^{\frac{3}{2}}} dt =$$

$$4 \times sgn(1+t) \int \frac{(1+t)}{(1+t^2)^{\frac{3}{2}}} dt = 4 \times sgn(1+t) \times \left(\int \frac{dt}{(1+t^2)^{\frac{3}{2}}} + \int \frac{tdt}{(1+t^2)^{\frac{3}{2}}} \right) =$$

$$4 \times sgn(1+t) \times \left(\int \frac{dt}{(1+t^2)^{\frac{3}{2}}} + \int \frac{1}{2} \frac{d(1+t^2)}{(1+t^2)^{\frac{3}{2}}} \right) =$$

$$4 \times sgn(1+t) \times \left(\frac{t}{\sqrt{1+t^2}} + C_1 - \frac{1}{\sqrt{1+t^2}} + C_2 \right) =$$

$$4 \times \left((t-1) \times \frac{sgn(1+t)}{\sqrt{1+t^2}} + C_3 \right) =$$

$$4 \times \left(\frac{t-1}{t+1} \times \frac{\sqrt{(1+t)^2}}{\sqrt{1+t^2}} + C_3 \right).$$

$$t = \tan\left(\frac{g}{2}\right) = \tan\left(\frac{x}{4}\right)$$

$$\int \left(1 + \sin\left(\frac{x}{2}\right)\right)^{\frac{1}{2}} dx = 4 \times \left(\frac{t-1}{t+1} \times \frac{\sqrt{(1+t)^2}}{\sqrt{1+t^2}} + C_3\right) =$$

$$4 \times \left(\frac{\tan\left(\frac{x}{4}\right) - 1}{\tan\left(\frac{x}{4}\right) + 1} \times \frac{\sqrt{\left(1 + \tan\left(\frac{x}{4}\right)\right)^2}}{\sqrt{1 + \tan^2\left(\frac{x}{4}\right)}} + C_3\right) = \left|\begin{matrix} \times \cos\left(\frac{x}{4}\right) \\ \times \cos\left(\frac{x}{4}\right) \end{matrix}\right. =$$

$$4 \times \left(\frac{\sin\left(\frac{x}{4}\right) - \cos\left(\frac{x}{4}\right)}{\sin\left(\frac{x}{4}\right) + \cos\left(\frac{x}{4}\right)} \times \sqrt{\frac{\left(1 + \tan\left(\frac{x}{4}\right)\right)^2}{1 + \tan^2\left(\frac{x}{4}\right)}} + C_3\right) =$$

$$4 \times \left(\frac{\sin\left(\frac{x}{4}\right) - \cos\left(\frac{x}{4}\right)}{\sin\left(\frac{x}{4}\right) + \cos\left(\frac{x}{4}\right)} \times \sqrt{1 + \frac{2\tan\left(\frac{x}{4}\right)}{1 + \tan^2\left(\frac{x}{4}\right)}} + C_3\right) =$$

$$4 \times \left(\frac{\sin\left(\frac{x}{4}\right) - \cos\left(\frac{x}{4}\right)}{\sin\left(\frac{x}{4}\right) + \cos\left(\frac{x}{4}\right)} \times \sqrt{1 + \sin\left(\frac{x}{2}\right)} + C_3\right),$$

where $C_1, C_2, C_3 -$ constants.