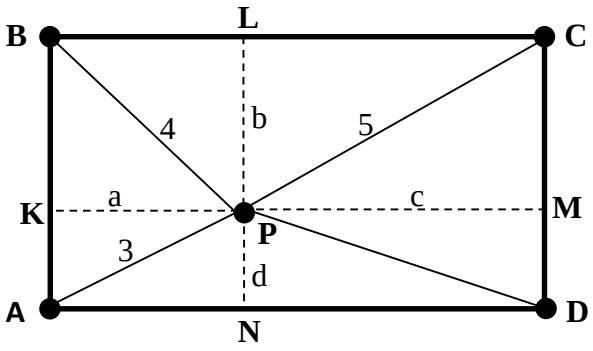
Answer on Question #45234 – Math - Geometry

P is a point in the interior of a rectangle ABCD. if P is joined to each of the vertices of the rectangle and the lengths of PA ,PB and PC are 3cm,4cm and 5cm respectively prove that $PA^2+PC^2=PB^2+PD^2$ and find the length of PD

Solution:



$$\begin{cases} a^2+d^2=PA^2=3^2=9 & (1) \\ a^2+b^2=PB^2=4^2=16 & (2) \\ b^2+c^2=PC^2=5^2=25 & (3) \\ c^2+d^2=PD^2 & (4) \end{cases}$$

From (1) + (3) and (2) + (4) we will obtain:

$$\begin{cases} PA^2+PC^2=a^2+d^2+b^2+c^2 \\ PB^2+PD^2=a^2+b^2+c^2+d^2 \end{cases}$$

From this, we can see:

$$PA^2+PC^2=PB^2+PD^2$$

So we can find PD

$$PD^2=PA^2+PC^2-PB^2=9+25-16=18$$
$$PD=\sqrt{18}=3\sqrt{2}cm$$